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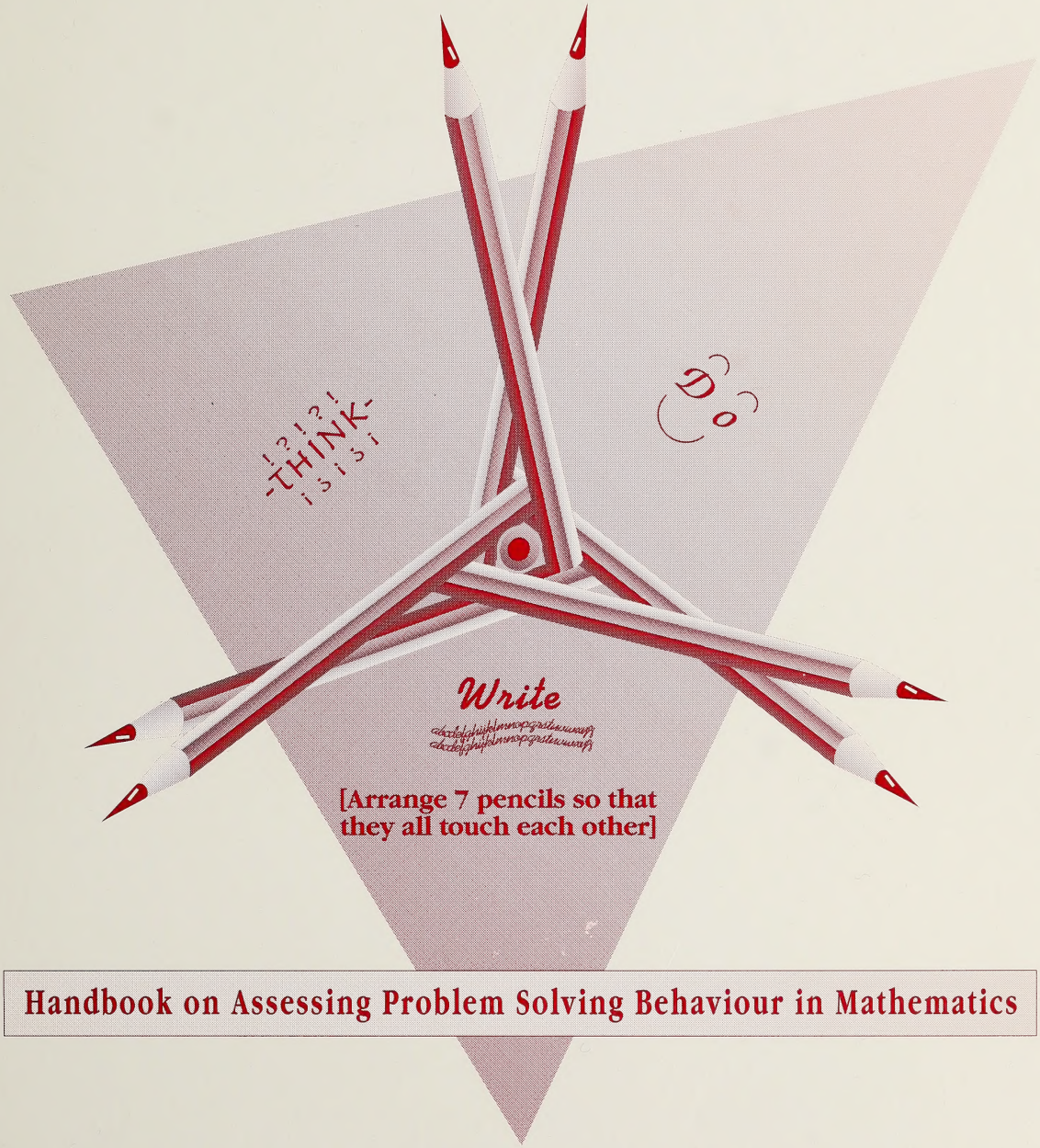
PROBLEM SOLVING PROFILES



Handbook on Assessing Problem Solving Behaviour in Mathematics

Educational Quality Indicators (EQI) Mathematics Project
Calgary Board of Education ▲ Calgary Catholic Board of Education ▼ Alberta Education

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Problem Solving Profiles

Handbook on Assessing Problem Solving Behaviour in Mathematics

Calgary School District No. 19
and Calgary RCSSD No. 1

Under Contract to Alberta Education
Edmonton, Alberta

Please Note

The views and recommendations expressed in this report are those of the researchers and not necessarily those of the Department of Education.

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[Students' written solutions were interpreted by a sample of the mathematics teachers and advisors of the project.]

PREAMBLE

The procedures described in this handbook were piloted in eight classrooms from September to December, 1991. The teachers of these classrooms (referred to as pilot teachers) were experienced mathematics teachers.

The pilot teachers attended two days of inservice which covered all aspects of the pilot process, from the philosophy of problem solving to filling out necessary project forms.

The pilot teachers recorded students' problem solving dispositions at the beginning and at the end of the pilot period. They administered six problems to their classes and each student response was assessed twice.

Four university graduate students, who were coached in "flexible interviewing" techniques, recorded both video and audiotapes of interviews they conducted with two students per classroom piloted during each problem solving session.

The pilot teachers documented observations they made of three students during each pilot problem assignment.

An outline of the activities and outcomes of the EQI-Mathematics project for each phase from 1989 to 1992 can be found in the Appendix of this handbook.

INTRODUCTION

In recent changes to Alberta mathematics curricula, the most important goal of instruction became the development of students' abilities to solve problems. Problem solving includes the acquisition of specific skills and processes as well as the development of positive attitudes towards the use of mathematical reasoning in many different life situations. These expectations have been accompanied by some change in teaching methods in schools, and intense interest in developing new methods for assessment.

The EQI-Math project was undertaken to identify effective non-traditional methods of assessing problem solving in mathematics. The goal of this handbook is to provide some guidance to teachers to help and encourage them to adopt these methods in their teaching. It is a handbook for all teachers since even experienced teachers need help with effective assessment because it is more complex than it appears.

Assessment is the act of acquiring and interpreting information about students for some stated purpose. In this case, the purpose is the improvement of learning and teaching of problem solving for both teachers and students.

The handbook outlines three methods of assessment — **observation, flexible interviewing and holistic analysis.** Each is presented in a separate section. However, they are not alternative methods and one cannot be chosen over the others. They focus on different aspects of the students' behaviour. **Observation focuses on the students' actions, interviewing focuses on their thinking and holistic analysis focuses on their written work.** All three methods are necessary components of the assessment model they define and must all be considered in exploring and assessing the students' understanding of problem solving.

The problem solving solutions presented in this handbook are consistent with the philosophy of the Program of Studies. This material has been reviewed by the Alberta Education Curriculum Department and, although it is not intended as a core resource, it does support the Alberta mathematics curricula.

▲ FOCUS ON ACTIONS ▼

Valuable assessment information about a student can be obtained from direct observation. In using this method, targeted students are watched carefully and their behaviours are noted and interpreted. Observation can range from nonstructured or informal, where one watches the student engaged in problem solving and gets an overall picture of the situation, to highly structured or formal, where each of the student's behaviour is recorded and the frequency of occurrences noted over a specified period of time. Teachers should try to function closer to the formal than the informal end of the continuum when they intend to use it for assessing the students.

To engage in effective and meaningful observation, the following guidelines should be considered.

1. Observation should be used to capture those qualities of a student's behaviour that cannot be more easily assessed using other methods. The focus should be on students' actions to assess attitude. As chosen students solve problems, either working individually or in a small group,

   watch them,
   listen as they talk to others and
   record your observations.

2. Observation should cover the following situations of a student's behaviour.
   i) Student participation and interaction with people — one's self, one's teacher, one's peers.
   ii) Student participation and interaction with tasks — the problems.
   iii) Student participation and interaction with objects.

The following are some of the observations made by a sample of experienced mathematics teachers (participants of the EQI project) that have been categorized under the three components to illustrate the nature of the observation for each.

I) Participation and Interaction with people.

Self	Peers	Teacher
talked to himself	listened to partner	asks to return to own desk
stretched	talks with partner	asks a question about
yawned	compares what partner has	a problem e.g. Does it mean
gazed into air	teaches "K" how to divide	by four?
looks around	brainstorming with others	Which way is East?
worked alone	joins in group discussion	Is this right?
looking down	discusses with partner	Is my logic correct?
leaning head on hand	works independent of partner	
hands on chin	corrects "J's" thinking	
frowning	initiated discussion	
smiling		
bites nails		
sits very thoughtful		

II) Participation and Interaction with tasks.

reading	rereads
writing	no marks on paper
drawing — stamps, a box, people, a rabbit, etc.	working continually
tries another set of two numbers	began working immediately
wrote out what he knew	records 8
checks graph	writing sentence to answer the problem
recopied problem	colouring
paused-started-paused-started	10 mins before writing anything

III) Participation and interaction with objects.

erasing	playing with paper
used calculator	sideways in desk
slouched in chair	looking over back of desk at floor
took watch off and put it on	using fingers (to count)
rubbed eraser between fingers	choosing pencil
dropped pencil	counts 14 cubes - connects into a train
kicks lunch bag	plays with unifix
looking at wall	pencil tapping
leaning over back of desk	

These components may not always be mutually exclusive. However, the goal is not to isolate them or develop distinct boundaries, but to make sure that they are all considered in the process. It is important to cover all three, not necessarily in the same observation, to avoid biased or conflicting assessment.

3. Observation should be linked to problem solving dispositions.

Problem Solving Dispositions

Motivated

- shows interest, curiosity, enthusiasm and inventiveness in doing mathematics*
- goes beyond the assignment
- reluctant to stop*
- works to a goal
- works until satisfied with solution
- works for a lengthy period using one strategy

Creative

- flexible in exploring mathematical ideas*
- tries alternative methods*
- invents own problems, formulates problems
- able to improvise in selecting materials to represent the problem

Confident

- belief in ability to use mathematics to solve problems*
- belief in ability to reason*
- not afraid to make mistakes
- belief that problems can be solved
- ability to identify and use necessary resources
- prepared to demonstrate for other students
- belief in ability to communicate ideas

Strategic Approach

- can state the problem in own words
- understands the meaning of words in the problem
- is able to draw a diagram that reflects the problem
- knows what information is needed to solve the problem and what information is extraneous**

Strategic Process

- uses organized and logical approaches
- takes time to stop and think
- notices and corrects one's mistakes**
- monitors one's thinking* – is reflective

* Terminology used in NCTM, *Curriculum and Evaluation Standards for School Mathematics*, 1989. p. 228

** Terminology used in Alberta Education, *Diagnostic Mathematics Program*, 1991, "Cognitive Processes."

These mathematical dispositions, identified by the EQI-Math project, should be used to:

i) frame the observations, i.e., to determine what aspect(s) of students' behaviour to focus on;
ii) interpret the students' actions.

The dispositions describe the type of behaviours that are associated with successful problem solving. Thus they should be used as a basis of determining whether a student possesses or does not possess these qualities. It is more useful to wait after a few observations before making interpretations. Interpretation after each observation could be misleading since a trend cannot be established with one or two observations. Only a few students (1 to 4) should be observed in a given problem solving situation.

The following are two cases of observations drawn from "anecdotal records" (records of observation and interpretation) prepared by the participating mathematics teachers of the EQI-Math project. The interpretations provided in these cases should be viewed as samples of how the mathematical dispositions can be used as a basis of assessing the students' actions and not as recipes to be followed thoughtlessly. Thus it is the process more than the product of the interpretation that is being illustrated here.

CASE 1

Observation #1

Read problem a couple of times.
Partner initiated discussion.
Listened attentively to partner.
Made no suggestions.
Discussion ended, but wrote nothing at first.
Tried to apply what partner told her.
Worked with ease for a few minutes.
Started to struggle with solution.
Crossed out things on her paper.
Stuck to same method.
Continued struggling on her own.

Observation #2

Read problem.
Started right away with a chart.
Checked and discussed with partner.
Partner did most of the explaining.
Sat very thoughtful.
Started writing.
Wrote a lot, worked intently.
Went to teacher, asked if logic was correct.
Returned to her seat.
Continued to write and work on solution.

Observation #3

Discussed problem with partner.
Listened while partner explained.
Started writing.
Kept writing without any stops.
Worked continually for about 1 hour.
Completed solution correctly.
Looked happy.

Observation #4

Read question.
Had brief discussion with partner.
Got right down to work.
Continued working on problem with no pause.
Completed after 20 minutes.
Got a right answer.
Seemed pleased with herself.

Interpretation

The trend established over this set of observations indicates that this student is very motivated. She shows interest by seeking help from, or engaging in discussions with her partner. She works for lengthy periods and works to a goal, i.e., until satisfied with her solution. She showed no reluctance in getting started and stayed focused throughout the process. There is no indication of creativity. She doesn't seem to be very confident in that she didn't take the lead in discussions with her partner, started her solution only after receiving help or suggestions from her partner and queried her teacher about her logic.

There isn't adequate information to draw conclusions about strategic approach and process since the observations did not focus on these qualities of the student's behaviour consistently. More information on what she talked about with her partner and what she was doing on her page is needed. If she was simply doing what her partner told her in a rote manner then she could be considered weak in these dispositions.

CASE 2

Observation #1

Read problem over and over again.
Had lost look on his face.
No discussion with partner.
Stared at paper.
Seemed afraid to write things down.
Continued staring for about 10 minutes before writing something down.
Wrote very little.
Kept looking around at others.
Had very little down on paper after 20 minutes.
Had no real strategy.
Did not do much.

Observation #2

Read question.
Stared at paper.
Started writing after about 5 minutes.
Long pause — staring, looking around, no writing.
Started to write again.
Seemed to struggle to find a solution.
Continued to struggle — wasn't sure what to do, which way to go.

Observation #3

Read problem over and over again.
No discussion with partner.
Started writing.
Stopped, sat and did more thinking.
Appeared lost.
Wrote again.
Continued working for about 15-20 minutes.
Stopped and closed booklet.
Was able to work out some solution.

Observation #4

Read question.
No discussion with partner.
Some time elapsed before he started to write anything.
Looked straight ahead for a few seconds.
Reread question.
Started writing after 5 or 6 minutes.
Started to find different paths on diagram.
Paused.
Started again.
Got lost in his counting, got confused but kept trying.
Completed question in 30 minutes.

Interpretation

This student seems to be somewhat motivated in that he is willing to try the problems, does not quit even though he encounters a lot of difficulties and tries to get to a solution on his own. There is no indication of creativity. Although working alone could reflect confidence, in this case, given the difficulties the student encountered and that he did not engage his partner in a discussion, it is more likely to be a case of lacking confidence and not knowing how to interact with his partner. His hesitancies suggest a fear to make mistakes. He also seems to be unable to identify and use necessary resources given the amount of time he spent gazing and looking around.

The trend over the observations most strongly shows poor strategic approaches because of the difficulties on what to do, the many readings of the problems, the staring, the thoughtless pauses, the random stops and starts, etc. In fact, this disposition seems to be the most significant contributor of the student's difficulties.

▲ FOCUS ON THINKING ▼

Several aspects of the problem solving process are not revealed by the written solution. Finding out what goes on in a student's mind as he/she solves a problem is crucial to effective assessment. Flexible interviewing is a process that can be used to get into a student's mind to gain valuable insights of the student's thinking and perspective of making sense of the problem solving process.

Flexible interviewing is an open-ended procedure intended to give the student a chance to display his/her natural inclination in solving the problems. The student leads the interview in that the interviewer asks questions based on what the student says or does to probe into or clarify the student's thinking. The interviewer does not tell the student what he/she should or should not be doing, during the interview. The focus should be on discovering unanticipated thinking processes or mental strategies used by the students.

Detailed flexible interviews can be time consuming, but teachers could engage in a brief sequence of interviews designed to clarify the students' understanding and discovering their thinking processes. However, regardless of the length of the interview, the following principles should be employed:

i) **Use open-ended questions [see examples below].**

ii) **Ask questions that follow up on the student's answers to reveal interesting aspects of the student's thinking.**

iii) **Elicit some reflection from the student to obtain further evidence concerning the methods of thinking he/she is using.**

iv) **Be less concerned with whether the student gets the right answer and more concerned with how the answer was obtained.**

The following is a sample of questions used by some of the interviewers in the EQI-Math project that resulted in meaningful explanations/responses.

1. **What made you think to try (or do) that?**
2. **What made you think of that?**
3. **Can you explain to me what you've just done?**
4. **Can you explain to me how you figured that out?**
5. **That's really interesting but I don't think I caught what you were saying, can you explain it again?**
6. **What does your answer mean?**
7. **What are you thinking?**
8. **Can you describe to me in your own words what the question is asking you?**
9. **That's interesting, how did you know to try ____ ?**
10. **Can you tell me what it is that you are going to have to do for this problem?**
11. **How do you know that _____ ?**
12. **Can you tell me what you mean by _____ ?**
13. **I'm just curious to know how you thought to do that?**
14. **How could you check that that answer is right?**

NOTE: Questions that had "why" in them did not get useful responses.
For example:

Q. Can you tell me why you said three or more ways?
A. I don't know.

Q. Why did you do that?
A. Probably because I am only good at counting by twos.

In general, "why" questions and questions requiring yes/no answers (i.e. closed ended questions) should be avoided.

The interviews conducted by the EQI project provide some useful situations that can inform us of the merits of interviewing or having students talk about their thinking. A sample of 6 of these situations follows.

CASE 1: Interview with a grade 5 student

Problem: How many five-cent and eight-cent stamps can you buy for exactly $2.43?

This interview allows us to see the unique thought process of this grade 5 student as she explains her mental computation in going from one number to the next using increments of 8, for example, 48 + 8 = 56.

Student: *.... I just imagined in my head, um like 'cause um because in the classroom we have a big board and it's a number — and um I usually look at it and I say to myself well, then we could go I know from 48 more is 48 and then if I imagine the board and I go up 8 more until you still have a 2 in that 40, and then you go up another 46 'cause 2 plus 4 is 3....*

Ok, 40 up another 8 is 48 and then you know there's still 2 in the forties left and then you go up 4 more and you get a 6 but it's in the fifties now....

Just keep going like that. ...

On the surface, this student may be perceived to not be making any sense and dismissed by teachers who are not willing to listen to understand the student's way of thinking. The student is actually using a mathematically sophisticated process to deal with simple addition. She is using the associative property of addition to work in "tens". So, 48 + 8 becomes 48 + (2 + 6) = (48 +2) + 6 = 50 + 6 = 56.

In her head, she does not go through these formal steps, but simply pictures that to add 48 and 8, 48 needs 2 more to complete the 40's and the remaining 6 should be in the 50's to get 56.

If this student had made an error in her computation on her worksheet, correcting her in terms of a more traditional approach may only confuse her. Thus having her explain her mental process provided useful information to help her, when necessary, in a meaningful way. In general, this situation suggests that we cannot always assume that students' thinking are simplistic or follow a traditional pattern.

CASE 2: Interview with a grade 2 student

Problem: **Billy had 14 marbles. He gave one-half of his marbles to Tom. How many marbles does Billy have left?**

This interview with this grade 2 student shows the importance of letting the students read the problem aloud, if they encounter difficulties in starting the solution, to assess the situation.

[In the following *S* = student, I = interviewer]

S: *(reads) Billy had 14 marbles. He gave one himself to his marble to Tom. How many marbles does Billy have left?*

I: Let's look back at the middle sentence again. Read this again.

S: *He gave one himself.*

I: Now let's look at that word again.

S: *Help?*

I: That's close, that's half.

S: *Half, he gave one-half to his, he gave one-half of his marbles to Tom.*

I: ... now read the whole thing again.

S: *Billy had 14 marbles. He gave one-half to of his marbles to Tom. How many marbles does Billy have left?*

The student was initially unable to solve the problem correctly not because of a lack of understanding of the problem, but difficulty in recognizing one word. She demonstrated this by immediately and quickly solving the problem once the word was corrected.

S: *It's um, it's asking you, like, Billy had 14 marbles. Then he gave one of his halves to Tom so he is asking you how much do he have, does he have left of this marbles but not counting Tom's...*

7 + 7 is 14, right?

So he gives one-half to Tom, so there is 14 marbles ...

So he gave one-half, so he gave 7 marbles to, he cut it off right here, he gave this to his friend and then would have seven marbles left ....

It should be noted that the meaning of the word "half" was not explained by the interviewer, neither the meaning of the problem, when the student could not identify the word. It should not be assumed that if a student makes a reading mistake, the word or the problem should be explained. Instead only the misread word should be corrected and the student allowed to interpret the problem on his/her own before further intervention.

In this case, the interview allowed us to separate difficulty with language from difficulty with the problem solving process to better assess the latter.

CASE 3: Interview with a grade 2 student

Problem: Show how many ways you can put 7 cookies in 2 cookie jars.

This interview with this grade 2 student reveals the student's level of thinking.

S: *... You can put three cookies in one and four cookies in the other; or you could put two in one and two in the other...*

(*) Oh, you could put three in this one, three in this one, like one apart, and put one piece in one jar.

And you can take two cookies, put it in here, and take five cookies and put it in here.

Take six cookies, put it in here, put one cookie in there.

Or seven in this one and none in that one.

The student recorded all of these possibilities diagrammatically and numerically except (*). The interviewer asked him to explain this possibility.

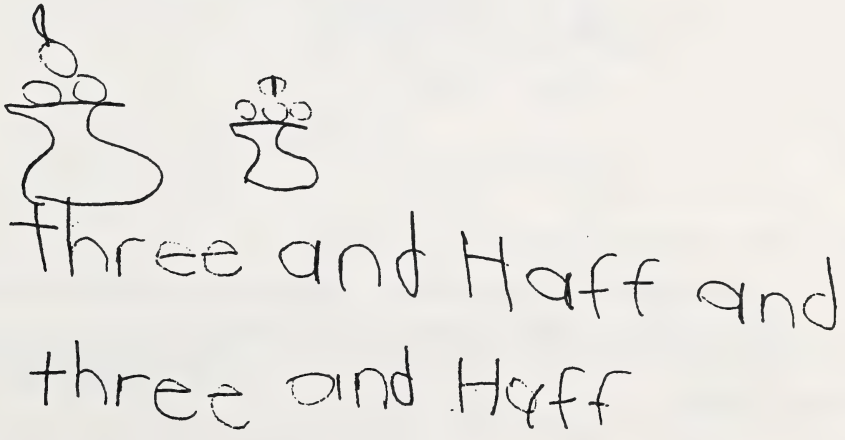
S: *You could put 3 in here and 3 in there, then break them in half and then put the other 2 pieces in there.*

I: Could you put that one down too?

S: *Three ... whoops, um I don't know how I can explain this one.*

I: You can write it in words if you want to...

He did the following:



This interview allowed us to capture processes one may not associate with this student's grade level. This student was able to conceptualize the problem beyond whole numbers (i.e. 3 1/2), but lacked the numerical symbolic knowledge to record it. Without the interview, this might not have surfaced and both the student and teacher would have failed to validate this creative thinking.

CASE 4: Interview with a grade 2 student

Problem: **Sally, Tina and Mary were trying to line up at the water fountain. Show how many ways the girls could line up if Mary always had to be first?**

This interview shows how this grade 2 student uses his own experience and vocabulary to understand the problem.

(...)

S: *... um, let's see here...Am I asking to budge somebody?*

I: I don't understand what you said.

S: *By asking to budge somebody, they could be first.*

I: Oh, you think that they're butting in you mean.

S: *Yuh.*

The student experienced difficulty because he did not know how to show the girls "budging". After further discussion and some guidance from the interviewer, he finally was able to represent the "budging" as follows:

He was then able to solve the problem using the diagram.

S: *... She could go from back here and budge, and then go to there. And there's another way ... and then just budge that person. There's two ways...*

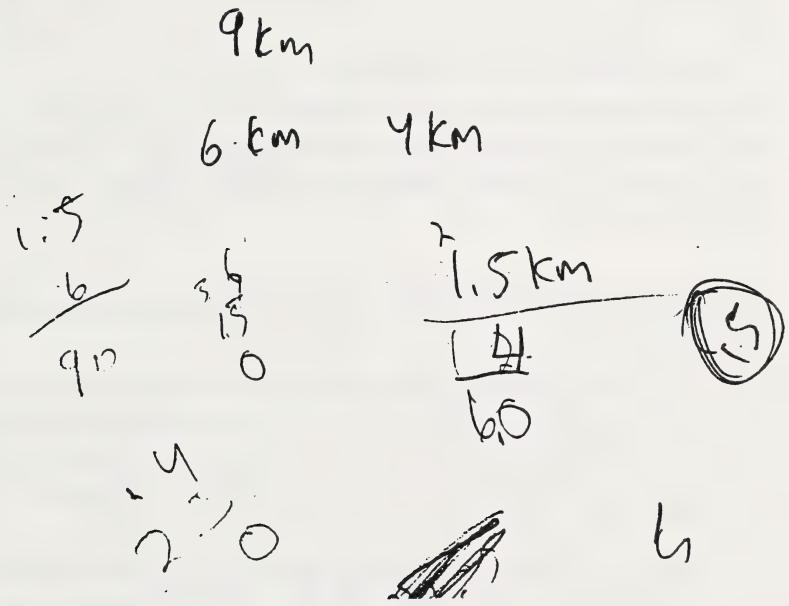
In this case, the interview informed us of the language that was meaningful to the student to help him solve the problem.

CASE 5: Interview with a grade 8 student

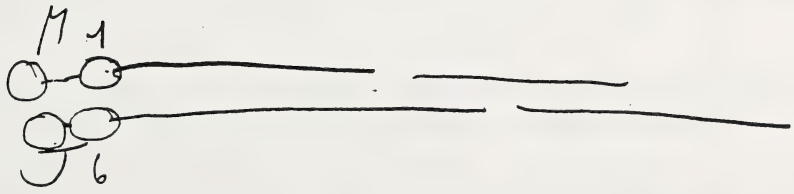
Problem: Jennifer and Mandy like to ride bicycles. Jennifer can ride 6 km in the same time Mandy can ride 4 km. The girls want to ride 9 km to Fish Creek Park and arrive at the same time. How much of a head start should Mandy have?

Student's work:

A.



B.



In this interview the student read the problem, took a few seconds, then said: I think it's 1.5.

I: How'd you get that?

S: Ok, well, I multiplied 1.5 x 4, so that's that. Ok. And I got 6, which is that. And then I timesed that and I got 9. Just a sec.

This response is part A of her written work — a "proof" of 1.5, which doesn't work. The interviewer asked if she could illustrate her answer by drawing a picture. She did this in part B of her written work with the following explanation.

S: *They're both here, hey, this is Mandy and this is the other one. And say she's going 4 km and she's going 6, like that, eh? She's slowly gettin' away farther and farther, so like if she went like, if she went 4, she would get this, then she would get that, but she would be here when she starts, so she would like be ahead. She'd get to there. She would go 4, she would go here, and this one goes to there, she'd have, Mandy'd have a head start all the time.*

(...)

I: You got 1.5 easy. How'd you get that?

S: *Well, I just kind of looked at it and I said, "that's gonna be 2 ahead of this — they're both even numbers ...and to get that you...so I was just thinking, these are both even numbers and this is an odd number and the difference between these is 2 and maybe...to get it there."*

(...)

S: *The answer, I think, is 1.5. I got this by seeing the differences in the numbers and multiplying to check.*

The interview shows that the student understood the context of the problem but not how to translate it mathematically. So she ignored the context of the problem and simply worked with the numbers, seemingly convinced that this was logical even though it did not check out. Even with coaching from the interviewer, she wasn't able to make the necessary connection between the problem situation and the maths. But she was "happy" with her solution.

Without the interview, this difficulty may not have been uncovered since written solutions cannot provide what the student is not aware of. Talking with the student will help the teacher to see such inconsistencies or conflicts in her problem solving approach and be in a better position to provide more meaningful help.

CASE 6: Interview with a grade 10 student

Problem: 525 can be written as the sum of consecutive numbers.

A. Find a set of consecutive numbers that will work.

Student's work:

A:

75 → 91 = 546
70 → 77 = 518
50 → 59 = 545
25 → 40 = 520

B:

1 + 525 = 526
2 + 524 = 526
3 + 523 = 526

C:

$\frac{525}{2} = 262.5$

approx ½ of 525 = 262

262 + 263 = 525

This student describes the problem as "confusing" and the way she felt after trying to solve it as "frustrated". The interview allows us to see why the student made these assessments.

The student's written work would seem to contradict her assessment given the brevity, neatness and well structured lay out of it, including a correct answer. But what is lost on the page are the human factors of the problem solving process, the struggle and effort the student went through or exerted to arrive at a solution. This is similar to the way students do not see the struggle of others, particularly the teacher, when thought processes are not shared, but a tidy solution is presented on the blackboard and the student is led to believe that problem solving is effortless and magical.

This interview also shows that the linear flow of the solution does not reflect the actual flow of the student's thinking, i.e. one does not necessarily get to a solution in a direct, linear manner. Finally, this interview provides the student's rationale from the student's perspective for her process.

The following are excerpts from the interview.

S: *(after reading problem) I'm just wondering how many numbers this is going to take and where to start.*

Well, since it's asking for consecutive numbers, you just try. I think I will start with the one plus two plus three and see where that takes me.

Well, since it asks for the sum of the consecutive numbers, I wouldn't really know where to start, because you don't, I guess starting with one would be too small, because it would take forever, but another point, you don't want to start too high either.

Ok, I'm just wondering where to start. I could just start with 75, because it doesn't have a certain number of numbers that you can use. Like saying that you can only use five numbers. You can use as many as you want and I think that it makes it harder and a lot longer.

(Starts part A of written work.) *Ok, I'm going to start with 75, just add 76, that gives me 151, add 77, it gives me 228, add 78, 79, 385, add 80, 465, and 81. I don't think will work. Too large.*

(Why start with 75?) *Well I don't want to start too low because I thought it would take too long, like starting with one plus two plus three and so on. ... It's not exactly half or anything along the lines of half of 525. I just started because I didn't want it too large or too small.*

Okay, well I'll just write down what I have, starting with 75 and going to 81. I think it gave me 546, so from here there are so many different numbers that you could use.

(Switches approach.) *Ok, well I'll just see approximately what half of 525 would be. It would be about 260. I could start with that and see what happens. It would be too large though, because 260 plus 261 would equal 521 and that wouldn't work because we are using the two largest numbers and in a question like this, it looks so simple when I first look at it, but the longer you work on it the longer it takes.*

(Switches back to initial approach.) *Well, I'll try it with 70, because 546 was closer to 525 than what I just got. (...) gives me a larger number.*

(...) so maybe I will start with something even smaller. I'll start with 50 just to see what I come up with. (...) That's still too high but it's lower than I got with 75 through 81.

(...) So I guess if I start with something a little lower, maybe if I start with 25.

[After computing the outcome starting with 25, she next tried starting with 26 then 24.]

I still know there is something I can do that is a lot easier, but I can't figure out what it is. ...

Ok, we did something kind of like this before in math. (Switches to part B of written work.) *We were trying to find, I think, even numbers added together that were, that gave the sum of, I think, 101. Because, if you add one and then you add 525, that would give you 526. Then when you go on it's two plus 524 and that would equal 526, and just one more, three plus 523 equals 526. Now where to go from there I don't know, because this isn't exactly the same question.*

(Switches to part C of written work.) ...*If I started with half of 525 again, or approximately half, so half of 525 equals 262.5. Now if you had 262 and 263, that would give you 525. So that would be one set.*

In general, this interview informs us that we need to be aware of what students' thought processes actually look like to be sensitive to when and how to intervene in their thinking to provide help.

▲ FOCUS ON WRITTEN WORK ▼

HOLISTIC RESPONSE CRITERIA

EQI-Math
Pilot 1991-2

Preliminary Response

No attempt

Words do not reflect the problem:
- diagrams and drawings unclear or misrepresent the problem; and/or
- does not indicate which information is appropriate to finding a solution.

Begins, but fails to move toward a solution.

Partial Response

Begins the problem in a manner that could lead to a solution but fails to complete it;
- omits a significant part of the problem;
- makes a major computational error
- uses an inappropriate strategy for solving the problem.

Provides a solution, but explanation unclear;
- argument or explanation incomplete;
- diagram is inappropriate or unclear.
  (A correct answer with no evidence to support the solution must be further supported by an interview or else there is no basis on which to judge the process.)

Complete Response

Gives a complete response that includes a full solution with reasonably clear explanations;
- communicates sound supporting arguments;
- shows understanding of the mathematical ideas and processes.

Elegant Response

Gives a complete response with an "elegant" explanation;
- unusual insight displayed in solution;
- communicates strong supporting arguments.

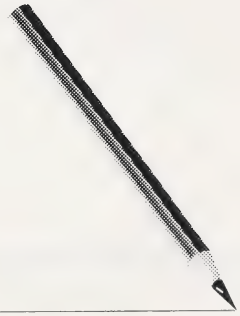
The EQI-Math project identified these Holistic Response Criteria as a basis of interpreting and assessing written solutions.

To provide some guidance to encourage teachers to use these criteria, an extensive sample of students' solutions (grades 2, 5, 8 and 10) with sample assessments prepared by experienced mathematics teachers are included in this section.

Given that the teaching of problem solving, particularly through the use of non-routine problems, is a new focus in the mathematics curriculum, it will require more experience with the teaching of it before more reflective assessments of students' work will surface. Consequently, the sample assessments presented here are not intended to be a prescription. They illustrate how a sample of experienced teachers, beginning to use the holistic response criteria, viewed the students' solutions. **Thus these samples provide a point of entry for teachers into the assessment process and are not intended to be a point of exit in terms of what an ideal assessment would look like.** The eventual goal is for teachers to see beyond the traditional structure (for example focus on mechanical and cognitive steps) of solutions to identifying, appreciating and validating the creative, intuitive and experiential thinking that are integral features of real world problem solving.

Teachers are encouraged to use the samples as benchmarks to help them to evaluate or reflect on what they have been doing, should be doing or will like to do. In general, teachers should use the samples to make more sense of their own situations, to question themselves about how they would react to the same or a similar situation, thereby becoming more aware of what they do or not do when assessing students' written work.

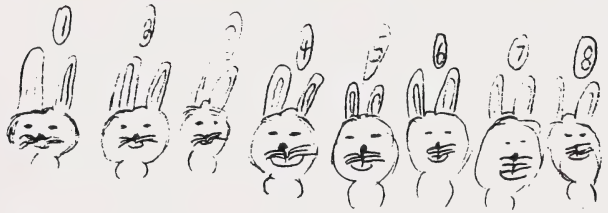
The sample assessments reflect significant variations in the perspectives used by the teachers. For example, the grade 2 samples reflect a personal reflective style of assessment, that is, a description of how the teachers viewed the students' processes. The teachers focused more on understanding what the students did as opposed to making judgements with respect to what they should have done. In contrast, the grade 8 samples reflect an impersonal style of assessment. Some of the teachers focused on making judgements of what the students should have done, with little or no attention to describing how they (the teachers) understood or viewed what the students had done. It is not being recommended that one perspective should be used over the other. Instead, teachers should try to consider and incorporate all meaningful aspects of these perspectives to effectively assess their students.



Grade 2

Mary Jane saw 8 rabbits.
How many ears did those rabbits have in all?

SHOW AND EXPLAIN ALL YOUR WORK



those rabbits ha-ae 16 ears in all

▲*Complete*▼

The formal numerical equation is missing but the pictures linked with the numbers shows the thinking behind the answer of 16 ears in all. The numbers 1 to 8 instead of 1 to 16 make me wonder if this student saw this as 8 two times and knew it was 16 instead of drawing 8 rabbits then counting the ears one by one.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

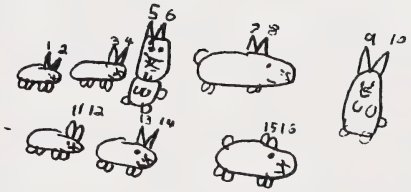
Discouraged *Satisfied* *Frustrated* *Happy*

Mary Jane saw 8 rabbits.
How many ears did those rabbits have in all?

SHOW AND EXPLAIN ALL YOUR WORK

eight and eight

1



▲*Complete*▼

The words eight and eight above the picture indicate the child understood the problem as needing eight twice. The single 1 line looks like a start to write 16 but didn't complete it. Even though there is not a formal math sentence, this child clearly knew what to do to solve the problem and demonstrate their way of thinking about it. The pictures indicate a justification of the answer 16 as opposed to seeing the problem as 8, two times.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

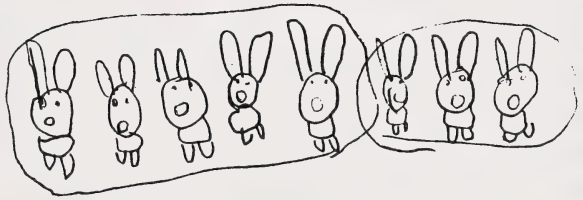
Discouraged *Satisfied* *Frustrated* *Happy*

Mary Jane saw 8 rabbits.
How many ears did those rabbits have in all?

SHOW AND EXPLAIN ALL YOUR WORK

16

16 it is 16. 10+6=16



▲*Complete* ▼

The work the child presents here demonstrates a different representation of the problem than most. The 16 at the top indicates after reading the problem the child knew the answer was 16. The 16 is written again and then we see words and pictures and an equation to justify the answer. The picture of 8 rabbits is drawn but instead of seeing 8 rabbits with 2 ears each, this child uses the picture to show us the understanding that the amount 16 is a group of 10 and 6 units. We also see the equation written in this form - 10 + 6 = 16. I feel this child easily understood the task, got the solution and the picture allows us a view into the mind and we see an understanding of the place value system.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

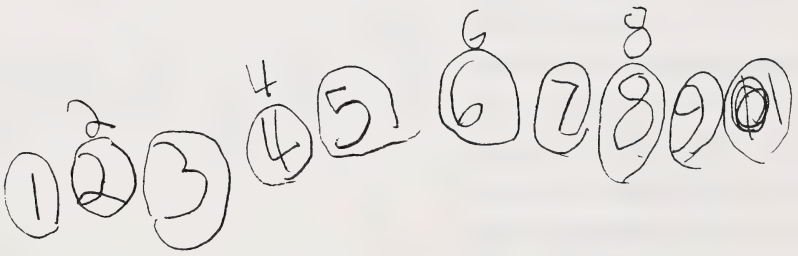
Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

I am a number less than 10.
I am also greater than 6.
You say me when you count by 2's.
Who am I?

SHOW AND EXPLAIN ALL YOUR WORK

8 is The ansr
I kawt id by 2s
and the problum sed
you will kumakros it



▲*Complete* ▼

The way this is written out is curious. The student has written at the top "8 is the answer," then in the explanation, summarizes the thinking as count by 2s and you will come across it. I am wondering if the number line at the bottom, which demonstrates more thinking than the written sentence, was done before the top part or after. The number line indicates to me more step by step thinking. As the learner read through the problem then the sentences does. So, depending which was done first, the number line assisted in working out the problem or was added to explain and justify the answer.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

I am a number less than 10.
I am also greater than 6.
You say me when you count by 2's.
Who am I?

SHOW AND EXPLAIN ALL YOUR WORK

It is 8

I cawntied by 2's and itsed it's less than 10 and it's greater than 6 it 1 than it must be 8

▲ *Complete* ▼

I feel the child was able to answer this problem easily, without actually having to write anything down. This was an activity in metacognition for this child, not in problem.

The writing to explain the work was a reflection on the process this child used to get an answer. The child clearly explained how he/she thought through the problem. It is interesting that the child said I counted by 2s first and it is last piece of information. This child read the whole problem and "saw" what the answer was.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

I am a number less than 10.
I am also greater than 6.
You say me when you count by 2's.
Who am I?

The answer is 8 I think

SHOW AND EXPLAIN ALL YOUR WORK

12345 6 7 8 9 10 -

6 7 8 9 10

2 4 6 8 10
tow

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

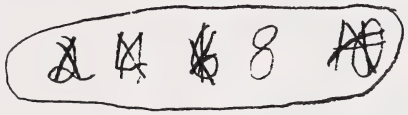
▲*Complete* ▼

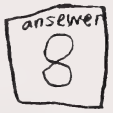
The circles, erase marks and the three different sets of numbers indicate to me this was a genuine problem to be worked through for this student.

It appears that the child worked through the problem in stages. Upon reading the first line, they wrote 1 2 3 4 5 no space 6 7 8 9 10. This was then erased and written 1 2 3 4 5 space 6 7 8 9 10. The space may indicate a beginning of a strategy to solve the problem greater than 6. The numbers 1 2 3 4 5 circled indicates in some way these are now not necessary. Greater plus 6 is circled in the problem and then we see the numbers 7 8 9 10. The child still has not realized the odd numbers are not involved. The 6 and 10 circled indicate the upper and lower limits are identified. Upon reading count by 2's, the child almost begins again. If they had built on the previous work, the 2 and 4 would not have been written. In the sequence 2 4 6 8 10, the 6 and 10 are circled to acknowledge parameters and the answer is discovered.

I am a number less than 10.
I am also greater than 6.
You say me when you count by 2's.
Who am I?

SHOW AND EXPLAIN ALL YOUR WORK





The quwestin say's she's less than 10.

And she's greater than 6.

And you don't say 7 when you caunt by 2's

so it has too be 8

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* (*Happy*)

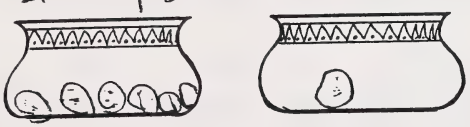
▲*Complete* ▼

The number sequence written down makes me feel this student read through the whole problem and realized it was unnecessary to record the sequence counting by 1's and necessary to stop at 10. The Xs indicate to me the student re-read the problem and eliminated the choices as information was presented. The 6 and 10 are crossed out twice which may indicate the student was reasoning and affirming the answer as they re-read the problem. The writing supports the symbolic work but begins as more of a re-wording of the problem than a description to explain. The last two lines justify the answer of 8.

This was a problem the child had to reason through. I would say this child did not "easily see" the answer.

Show how many ways you can put 7 cookies in 2 cookie jars?

I put 6 in one



and 1 in the uther one.

SHOW AND EXPLAIN ALL YOUR WORK

I put 3 in one and

I put 4 in the uther one.

I put 2 in one and 2 in the uther.

one.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

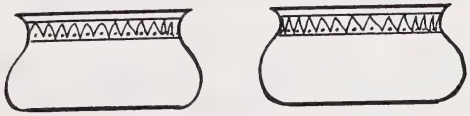
The way this child worked out this problem on the paper gives us an interesting look into how they, at this time in their development, see math. I would say this was a real "problem" for them to work at. They did not instantly see what the math involved was did the children in the other examples. This child needed to think through the problem. The child began with a successful strategy and continued to use.

Even though the child has drawn pictures, the wording gives the impression the child sees this as imagining actually putting the cookies in the two jars. The absence of the equation but use of numerals is interesting. For this child, the telling and, as shown here, the writing are the ways to communicate the solution.

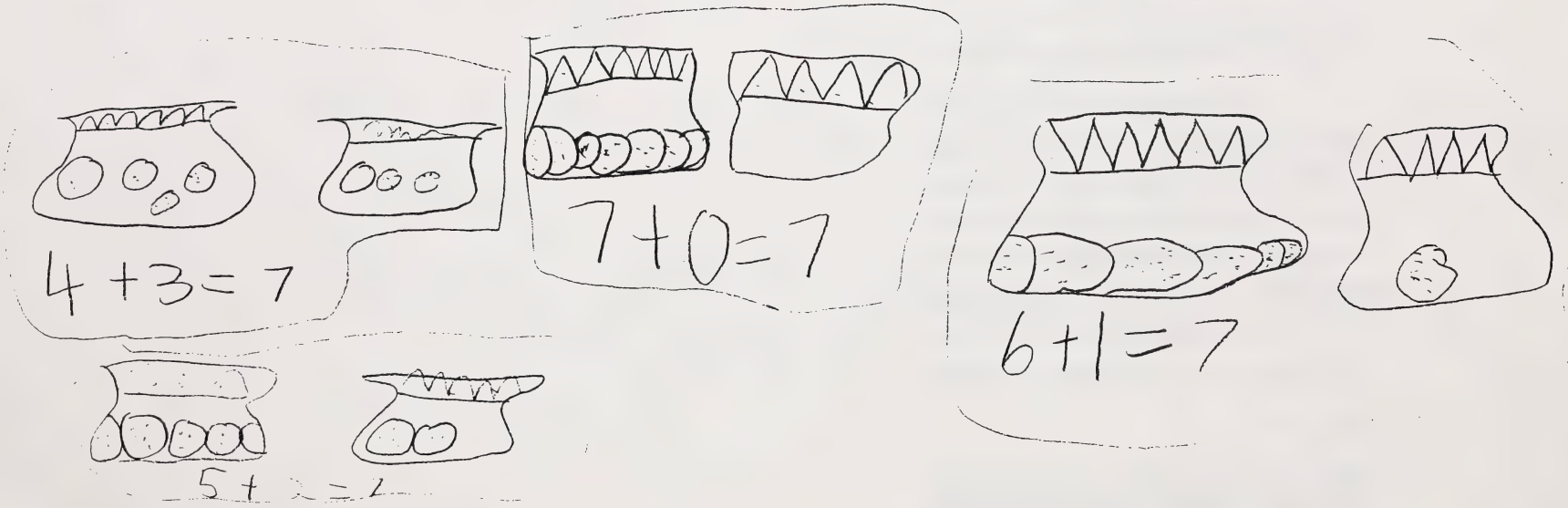
This child has not yet connected the full mathematical symbolism which we are trying to work towards to this problem.

Grade 2

Show how many ways you can put 7 cookies in 2 cookie jars?



SHOW AND EXPLAIN ALL YOUR WORK



▲ *Complete* ▼

This child's solution is complete even though all possible equations are not present. In an interview, I could quickly determine if they felt it did not matter which jar had which amount thus it would be unnecessary to show the opposites.

This child drew pictures and wrote equations to explain the pictures. There are no erase marks or things crossed out so I would say this child clearly understood what to do and the strategy they used successfully lead to a solution. This child recognized the problem as asking for combinations which equal 7.

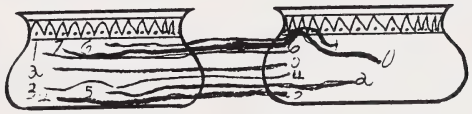
Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Show how many ways you can put 7 cookies in 2 cookie jars?



SHOW AND EXPLAIN ALL YOUR WORK

I put number's that make 7 and I put a line across to the number like this 5 and then I colored the line to the number like this.

Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Elegant* ▼

This child approached the problem differently than most children. This indicates to me a different level of understanding of the amount of 7. This child saw the problem as asking for the possible ways to make 7 and knew them so well did not have to draw out the pots and cookies. The student already saw the numbers that went together to equal 7. The pattern started (1-6, 2-5, 3-4, 4-3), then ran out of room, continued by going up to complete the pattern (5-2, 6-1, 7-0) demonstrates the way this child sees all possible combinations for 7. The written explanation lets us know he thought of the numbers that make 7 then drew the lines. It looks as though they were done in pencil and possibly, upon reflection of how the teacher would see this, realized the need to add colour to clear it up visually.

I feel this problem was not a "problem" for this child. He easily saw what was being asked and the challenge for him was to represent it clearly to the teacher. The child indicated the task was boring but felt satisfied when done.

Billy had 14 marbles.
He gave one-half of his marbles to Tom.
How many marbles does Billy have left?

SHOW AND EXPLAIN ALL YOUR WORK

So if Billy gave haf
of his marbles to tom
and Billy had 14 that
men's that he wood hav 7.

▲ *Unique* ▼

The child has personalized this problem. The two children in the picture are playing marbles and both have a set of marbles but the one is sharing his set with the other. The diagram shows the "action" lines of the marbles moving to Tom. The child's written explanation does not really give any insight into how he knew or figured out what 1/2 was. Because of the evidence of a more detailed picture, I would say the time was not spent on working on the problem but on the picture and writing. This child may have known, at an automatic level, that half of 14 is 7, so this may not have even been a problem to this child.

Circle **one** of the following words you would use to describe this problem:

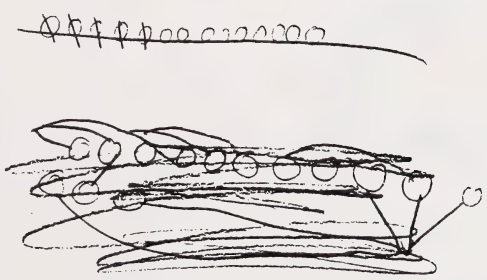
Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

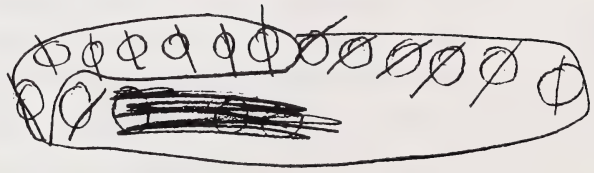
Discouraged *Satisfied* *Frustrated* (*Happy*)

Billy had 14 marbles.
He gave one-half of his marbles to Tom.
How many marbles does Billy have left?

SHOW AND EXPLAIN ALL YOUR WORK



i took my unfix cyobs and me and stacey K. went 1 1 2 2 3 3 4 4 5 5 6 6 until all of them were gone it add up to seven each.



▲*Complete* ▼

The explanation tells us the student worked with someone else and actually acted out the problem before recording it on paper. The strategy used was a sharing - one for you, one for me - until they were gone and when adding up each pile the answer was found. The diagrams show this child was using them to represent the actions the partners did. We see several attempts before one was a satisfactory representation of what happened.

Circle **one** of the following words you would use to describe this problem:

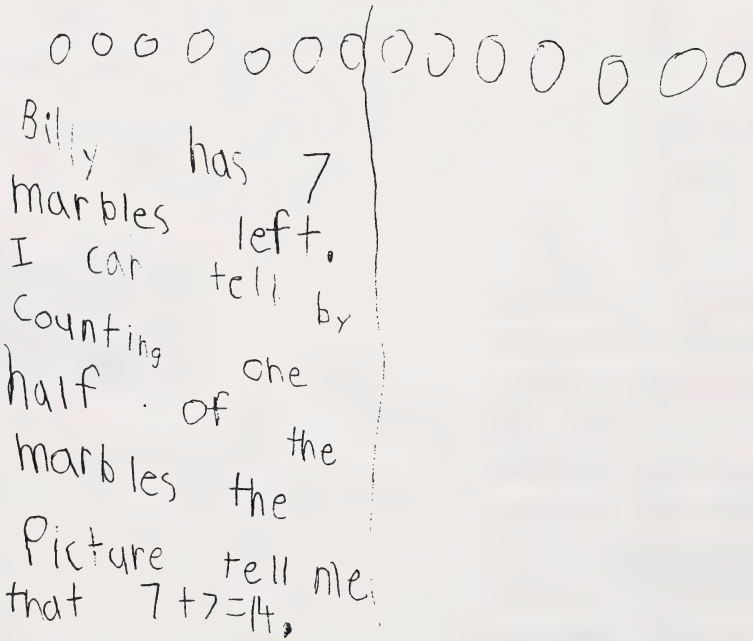
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Billy had 14 marbles.
He gave one-half of his marbles to Tom.
How many marbles does Billy have left?

SHOW AND EXPLAIN ALL YOUR WORK



▲*Complete*▼

The diagram with the line indicating half lets us know this problem needed some pictures to either help answer or justify an answer. The writing does not really tell us any strategy used. I would ask "How do you know when you have counted one half?" The symbolic equation tells me the child knew an equation was needed, but I wonder if they, in fact, used adding to solve it. This child may have known the basic facts well enough to see half of 14 is 7 and if so this was not a real "problem" for the child to work out.

Circle **one** of the following words you would use to describe this problem:

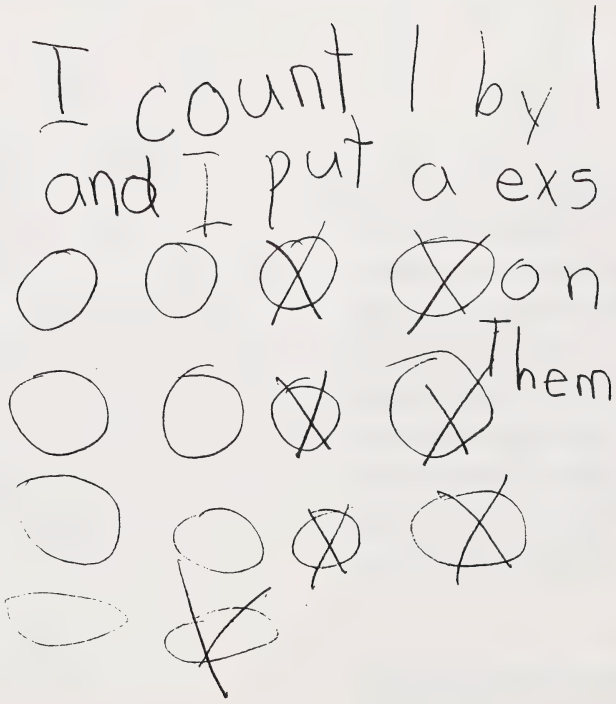
Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

Billy had 14 marbles.
He gave one-half of his marbles to Tom.
How many marbles does Billy have left?

SHOW AND EXPLAIN ALL YOUR WORK



Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

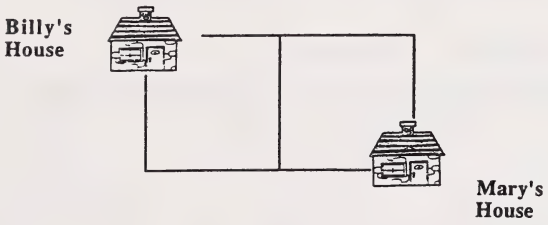
Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Complete* ▼

Even though this child does not include a numbers sentence, I feel this problem is complete. The answer is shown in pictorial form instead of symbolic form. The explanation leads me to think this child used a dividing process to solve this problem. The child drew 14 circles then shared by a "one for you, one for me" process until they were gone. The Xs indicate which person they belong to.

The lack of a symbolic answer may be that the child used the dividing process and did not know how to symbolize it.

Show how many different ways you could walk from Billy's house to Mary's house .



SHOW AND EXPLAIN ALL YOUR WORK

I read the Problem
I thenkt of a Plan
I Trou Piters
and thats all

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

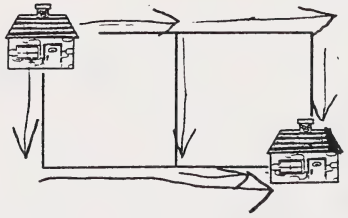
Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Unique* ▼

This delightful response needed comment. The child understood "different ways" to mean something very real in the situation but not how we intended it. The message to me here, is that this child needed to talk about this problem before attempting it. Some collaborative thinking may have produced different results. If this child did have some group talk prior to the problem, which is probably the case being problem #5, the child did not "hear" and may need some mediation in group work, to help focus on the aspect of "Understand the Problem." The writing on the second page indicates some knowledge of a problem solving model.

Show how many different ways you could walk from Billy's house to Mary's house .



HH 123

SHOW AND EXPLAIN ALL YOUR WORK

There are 5 Ways To go to Marys House

▲ *Partial* ▼

This student did not complete the problem to four routes but the one missing way is not a logical route because it goes toward the house then away from it. The marks on the side are interesting as they may indicate a tally record keeping system to keep track of the routes as they are discovered. The 1 2 3 is written possibly to better explain 3 lines so the lines are crossed out. The child clearly states his/her answer at the end.

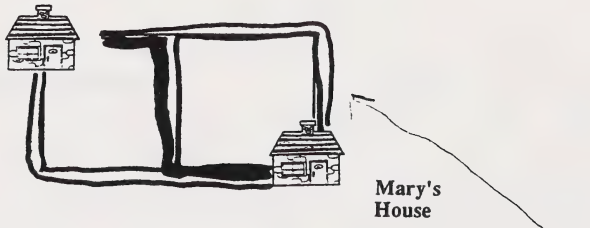
Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Show how many different ways you could walk from Billy's house to Mary's house .



SHOW AND EXPLAIN ALL YOUR WORK

I did four way's to Mary's house from Billy's house and these are the four way's.

▲ *Complete* ▼

This complete response demonstrates organized, thoughtful work. The student used different colours to represent the different paths. I wonder if the lines were drawn for the student to see each route or if the student did it for the teacher to see the answer clearly. Which ever way, the student seemed to prefer a visual representation of the problem to solve it. The answer is stated clearly through the writing.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Sally, Tina and Mary were trying to line up at the water fountain. Show how many ways the girls could line up if Mary always had to be first.

SHOW AND EXPLAIN ALL YOUR WORK

~~Someone~~

who ever wonted to ~~have~~ have a brik first shod of ben then faster

Suppose Sally always had to be first in line. Show the different ways the girls could line up.

SHOW AND EXPLAIN ALL YOUR WORK

How many different ways can the three girls form a line if any one of them can be first?

SHOW AND EXPLAIN ALL YOUR WORK

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Unique* ▼

I had to include this response to comment on. This response gives an insight into the person. This child saw the problem from another dimension than was intended. The mathematical knowledge cannot be directly commented on here but the response is rich in other ways. Be first, be faster to be first. If you are "trying" to line up and are not fast, others will get in front of you. This signals an approach to life.

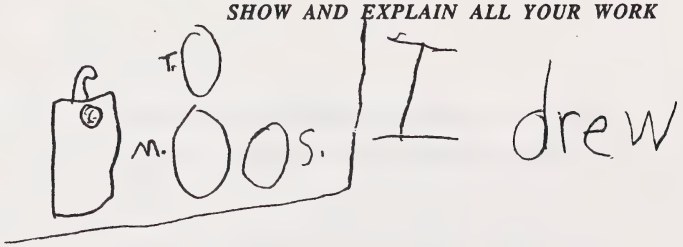
Another interesting element that comes from this non-math based answer is the importance of discourse when working through an activity. I wonder if this child would have answered in this way had he/she chosen to work with others. Some conversation, opinions, insights from other children into what the problem was asking might have influenced a different type of response.

Sally, Tina and Mary were trying to line up at the water fountain. Show how many ways the girls could line up if Mary always had to be first.

SHOW AND EXPLAIN ALL YOUR WORK

Suppose Sally always had to be first in line. Show the different ways the girls could line up.

SHOW AND EXPLAIN ALL YOUR WORK



I drew 3 Pictures my First Picture the line went strate my Secord Picture the line cerved my third it made two lines

How many different ways can the three girls form a line if any one of them can be first?

SHOW AND EXPLAIN ALL YOUR WORK

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Unique* ▼

This response looks differently at the interpretation of "line" than we would expect. We envision a straight, one behind the other, line but in a child's world these solutions are very real possibilities. A child standing beside another while waiting for a drink. Is it wrong, I would say not, just not conventional.

By not answering this in the conventional manner, the child tells me they have not yet experienced a situation where they need to systematically look at the placement according to 1st, 2nd, 3rd position. They saw the geometry/art use of line meaning straight or curved, and the real life situation at the fountain.

Sally, Tina and Mary were trying to line up at the water fountain.
Show how many ways the girls could line up if Mary always had to be first.

SHOW AND EXPLAIN ALL YOUR WORK

Sally Tina Mary

mts
mst

I will use thary first
arishle to doit. I fond
tow ways.

Suppose Sally always had to be first in line.
Show the different ways the girls could line up.

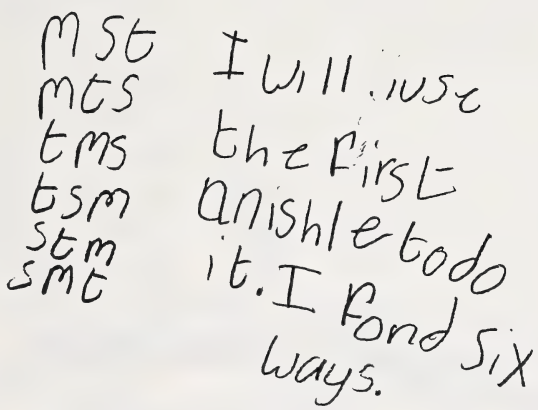
SHOW AND EXPLAIN ALL YOUR WORK

Stm
Smt

I will use thary
first arishle to do
it. I fond tow ways.

How many different ways can the three girls form a line if any one of them can be first?

SHOW AND EXPLAIN ALL YOUR WORK



Circle **one** of the following words you would use to describe this problem:

Boring *(Challenging)* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *(Satisfied)* *Frustrated* *Happy*

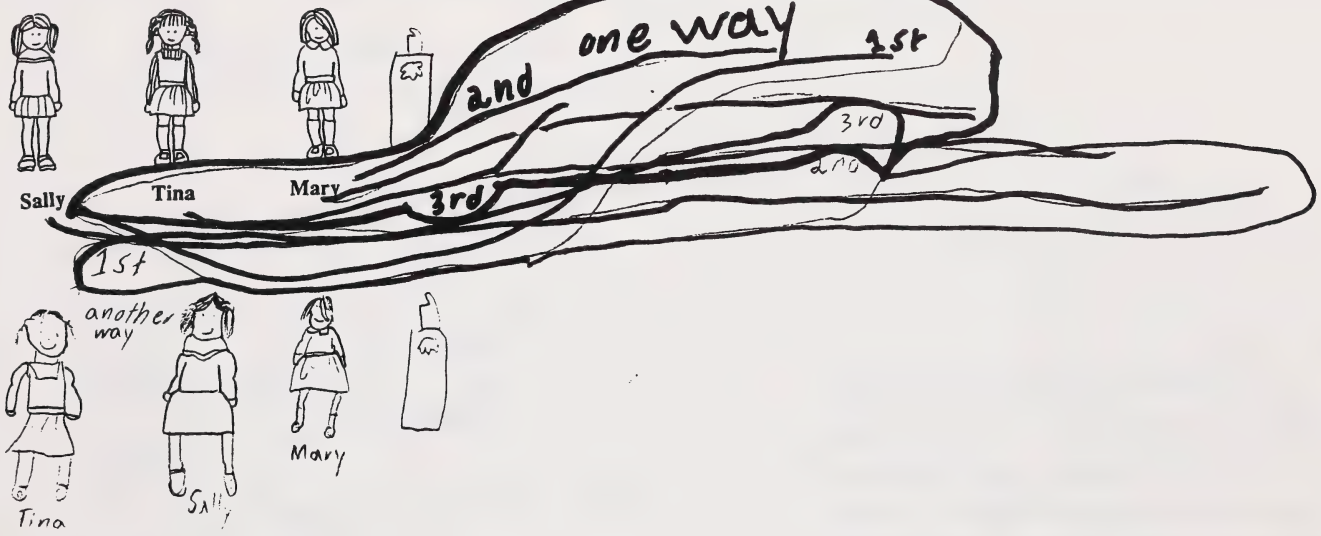
▲ *Complete* ▼

This child has systematically completed all three components of this problem. As indicated in their writing, they used the first initial to do it. The organization of the initials in a line facilitated seeing a pattern. I would say this child saw these as almost three separate problems and completely answered all three. The sentences state how many ways for each section which nicely completes the answers.

Grade 2

Sally, Tina and Mary were trying to line up at the water fountain. Show how many ways the girls could line up if Mary always had to be first.

SHOW AND EXPLAIN ALL YOUR WORK



~~fo~~ In the purple circle is one way for Sally to go first and in the green circle is another way. And these two are two way for Mary to go first

Suppose Sally always had to be first in line. Show the different ways the girls could line up.

SHOW AND EXPLAIN ALL YOUR WORK

How many different ways can the three girls form a line if any one of them can be first?

SHOW AND EXPLAIN ALL YOUR WORK

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Unique* ▼

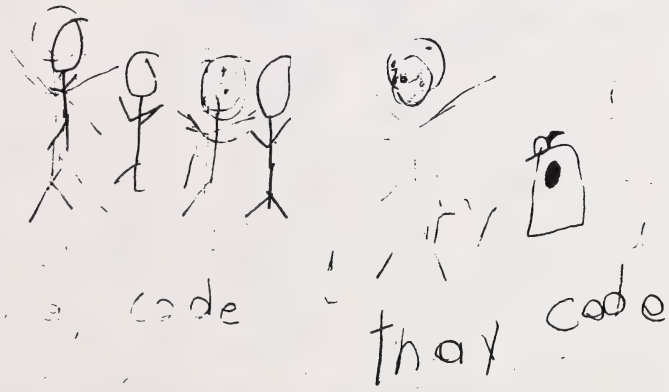
This is a wonderfully creative response. Each of the three sections build on the previous one. The child used the printed pictures of the girls and names as one of the solutions then drew the second below, even adding the details of the dresses. The coloured lines are the solution to the second part of the problem.

The written explanation explains the lines. The third part is interesting because I see it as focusing on the "different" ways as meaning different from what you have already shown us. The child used the previous solutions as being four of the six possible ways and drew the two remaining ways.

Sally, Tina and Mary were trying to line up at the water fountain. Show how many ways the girls could line up if Mary always had to be first.

SHOW AND EXPLAIN ALL YOUR WORK

Sally Tina Mary



Suppose Sally always had to be first in line. Show the different ways the girls could line up.

SHOW AND EXPLAIN ALL YOUR WORK

How many different ways can the three girls form a line if any one of them can be first?

SHOW AND EXPLAIN ALL YOUR WORK

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Elegant* ▼

Here is the elegant response to the problem! Perhaps not from a mathematical view but certainly from a true-to-life, living-in-this-world view. This child really worked at this. The erase marks indicate different positions from the fountain and the final solution makes me think this situation is never a real problem for some kids. I would now know who to call over when a child needs a friend! Can we ever see assessment in isolation of "being"?

Judith is thinking of 2 mystery numbers.
One number is 18 less than the other.
Both are two-digit numbers that are divisible by 3 but not by 6.
Show how you found these mystery numbers.

SHOW AND EXPLAIN ALL YOUR WORK

first I thoght of 12 and 27 bout that cann't be it because that is only 15 less than it. So I added 3 to 27 bout that is divisible of 6. So I added 3 to 12 and 30. that = 15 and 33.

81 99
75 93
69 87
63 81
57 75
51 69
45 63
39 57
33 51
27 45
21 39

15 36

15 - 3 = 12
3 - 15 = 18

$6\overline{)33}$ R3 — 30, 03, 03

$6\overline{)15}$ R3 — 12, 03, 03

$6\overline{)18}$ — 18, 00

With 15 and 33 you skip add 6 and you will get the answer right up to 99

With 15 and 33 you add 6 and you can get a nother divisible of 3 not 6. if you add 3 you will get a divisible of 6 and 3

▲ *Elegant/Complete* ▼

The student thought of two mystery numbers but realized the difference between them was not 18. He added 3 to those guesses for another possible solution but found they were divisible by 6. By adding 3 more to each, the student discovered a solution. He realized by adding 6 to each mystery number, he could find all the mystery number pairs to 99.

This student's response is more than "complete" because the student gave all possible solutions but the response is less than "elegant" because the supporting argument was not complete.

Circle **one** of the following words you would use to describe this problem:

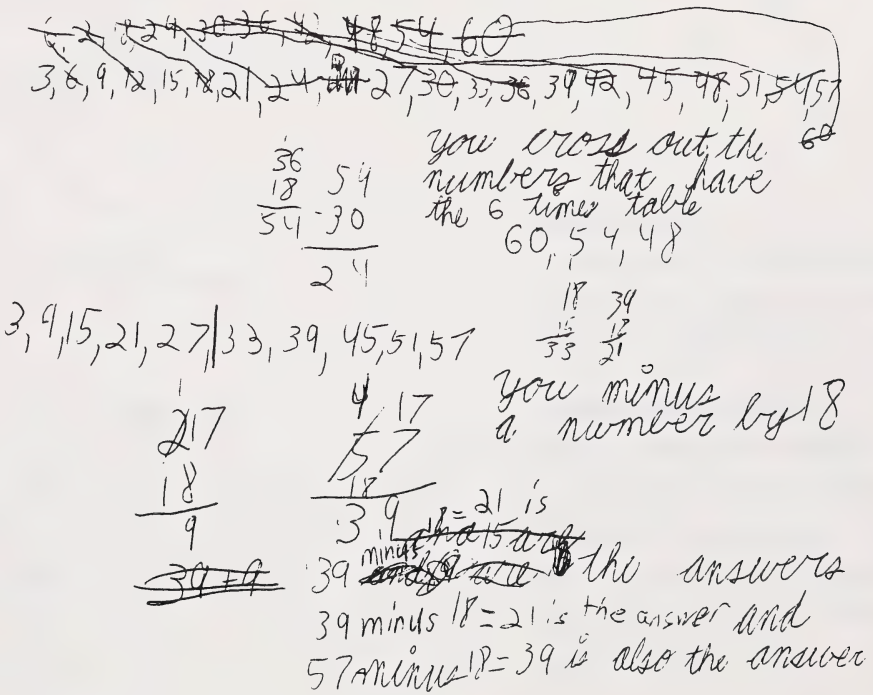
Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* (*Happy*)

Judith is thinking of 2 mystery numbers.
One number is 18 less than the other.
Both are two-digit numbers that are divisible by 3 but not by 6.
Show how you found these mystery numbers.

SHOW AND EXPLAIN ALL YOUR WORK


3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60
you cross out the numbers that have the 6 times table
60, 54, 48
36
18 54
54 -30
24
3, 9, 15, 21, 27 | 33, 39, 45, 51, 57
217
18
9
39 minus 18 = 21 is the answer
57 minus 18 = 39 is also the answer
you minus a number by 18

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

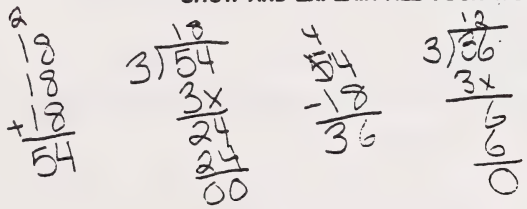
▲*Complete* ▼

The student wrote multiples of 3 and 6 and eliminated all multiples that were divisible by 6. The student rewrote the remaining possibilities then subtracted 18 from each to find the mystery pairs. S/he discarded the number that was not a two-digit number. Two possible solutions were stated with accompanying computation shown.

The student recognized common multiples of 3 and 6 and eliminated multiples of 6. Also, the student used the relationship of the difference of 18. The student's written response clearly indicates and sequence in development of his understanding.

Judith is thinking of 2 mystery numbers.
One number is 18 less than the other.
Both are two-digit numbers that are divisible by 3 but not by 6.
Show how you found these mystery numbers.

SHOW AND EXPLAIN ALL YOUR WORK



First I put 18 x 3 and I got 54. Then I checked to make sure that it was possible for me to divide it by three. When I discovered it was I thought that this was one of the mystery numbers so I put 54 - 18 it turned out to be 36 and I checked to make sure I could divide that by three and I could so I think that the two mystery numbers are 54 and 36.

Partial
- neglected to see that it was divisible by 3 but not by 6

▲ *Partial* ▼

The student recognized the need for a two-digit number and that the number had to be divisible by 3. There was not full understanding of where the difference of 18 was to be used. The student neglected to test that the mystery numbers were not divisible by 6.

The student began the problem with possibilities leading to a solution, however, but neglected to eliminate the multiples of 6.

Circle **one** of the following words you would use to describe this problem:

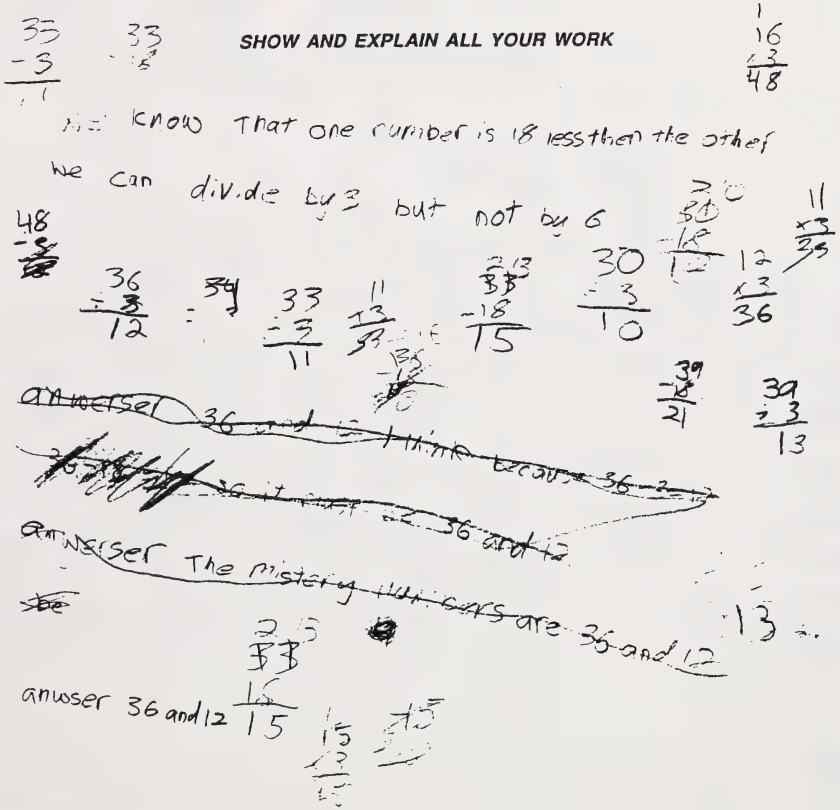
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Judith is thinking of 2 mystery numbers.
One number is 18 less than the other.
Both are two-digit numbers that are divisible by 3 but not by 6.
Show how you found these mystery numbers.

SHOW AND EXPLAIN ALL YOUR WORK


We know That one number is 18 less then the other
we can divide by 3 but not by 6
answer 36 and 12
answer The mistery numbers are 36 and 12

▲*Preliminary*▼

The student recognized the difference of 18 between the two mystery numbers and focused on the difference only. S/he began by dividing by 3 but then used numbers not divisible by 3.

The student began by restating the problem, but used only part of the information and was misdirected from a solution.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Dale was using a stencil to make numerals
for the art work display at school.
There were 100 pieces of art to be numbered in order
from 1 to 100.
How many 4's will Dale have to make?

SHOW AND EXPLAIN ALL YOUR WORK

4, 14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47,
48, 49, 54, 64, 74, 84, 94.

First we started on the number 4
and went up ten so we were on 14.
After we just went up ten and
so on. Then when we went up
to 94, we went back to the forty's
and did 41, 42, 43, 45, 46, 47, 48, 49. After
we went back just to check over
and we thought that there was
19 answers but we saw that 44
had two fours so the answer was
20 because the question says "How
many 4's will Dale have to
make?" not "how many numbers will
have a four in them?"

▲ *Elegant* ▼

The student began with the number 4 and counted by tens up to 94 to find all possible 4's in the ones, then went back to the numbers in the 40's to find all possible 4's in the tens. The student examined the number 44 and realized a possibility of one more 4. Student shows and communicates a strong supporting argument in restating the problem as "how many 4's" rather than "how many numbers will have a 4". Insight into the use of mathematical ideas is shown in the explanation.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Dale was using a stencil to make numerals
for the art work display at school.
There were 100 pieces of art to be numbered in order
from 1 to 100.
How many 4's will Dale have to make?

SHOW AND EXPLAIN ALL YOUR WORK

4 14 24 34 40 41 42 43 44 45 46 47 48 49 54
64 74 84 94

first we started at four
then we conted by 10's
till we got to 40 Then
we counted 41, 42, 43, 44, 45, 46, 47,
48, 49, then we started at 54
and counted by tens till
we got to 94
dale neded 20, 4 to get to
a hundred

▲*Complete*▼

Student began with the number 4 and counted by 10's to 34, then began at 40 and counted possible solutions to 49. The student continued with 54 counting by 10's to 94. The student then counted all 4's in the numbers written.

The student gave a complete explanation and communicated a sound supporting argument.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Dale was using a stencil to make numerals
for the art work display at school.
There were 100 pieces of art to be numbered in order
from 1 to 100.
How many 4's will Dale have to make?

SHOW AND EXPLAIN ALL YOUR WORK

1 answer

1. Dale is using a stencil to make numerals
2. He needs to number 100 pieces of art
3. We have to know how many fours there are from one to one-hundred

I think Dale wants to know how many fours there are in 1 to 100, I am. It has to be how many fours like (44) (they are both fours or (8) there are two fours in 8 but Dale wants to know how many fours he is going to make. And am going to do this:

4, 14, 24, 34, 44, 54, 64, 74, 84, 94

He is going to have to make 11 fours

▲ *Partial* ▼

The student began the problem correctly but got sidetracked by using a multiple of 4 but realized the error. The final solution, however, omits all of the numbers in the 40's except for 44, a significant part of the problem.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Dale was using a stencil to make numerals
for the art work display at school.
There were 100 pieces of art to be numbered in order
from 1 to 100.
How many 4's will Dale have to make?

SHOW AND EXPLAIN ALL YOUR WORK

4X1=4
4X2=8
4X3=12
4X4=16
4X5=20
4X6=24
4X7=28
4X8=32
4X9=36
4X10=40
4X11=44
4X12=48
4X13=52
4X14=56
4X15=60
4X16=64
4X17=68
4X18=72
4X19=76
4X20=80
4X21=84
4X22=88
4X23=92
4X24=96
4X25=100

Dale will 25 4s in his display
because 4X25=100. and 4÷100=25

$$4\overline{)100}\quad 25$$
8
20
20
0

▲ *Preliminary* ▼

This student misinterpreted numeral for number and found the number of multiples of 4 in 100 rather than how many times the numeral 4 occurs between 1 and 100.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* (*Happy*)

Martin counted the marbles he had collected.
He counted more than 40 but less than 70.
When he put the marbles in groups of 5 he had 1 left over.
When he put them in groups of 4 he had 1 left over.
When he put them in groups of 3 he had 1 left over.
How many marbles did Martin collect?

SHOW AND EXPLAIN ALL YOUR WORK

8
5|41
-40
01

10R1 13R1 12R1 12 12 15 20
4|41 3|41 5|61 4|51 5|61 4|61 3|61
-4 -3 -5 -4 -5 -4 -6
01 11 01 11 11 21 01
-0 -9 -0 -8 -10 -20 -3
1 02 1 3 01 01 1

What I did is I started with 41 since its more than 40 and divided it by five it worked out so I divided it by 4. when I was done it worked out so I tried dividing it by 3 it didn't work out. So I went to 51 were I finished it had worked out. So I divided it by 4 when I was done it worked out. So I went and divided it by 3 it didn't work. So I went to 61 were I finished it had worked out. So I divided it by four it worked out. So I divided it by 3 it worked out. So Marten collected 61 marbles.

▲*Complete* ▼

The student began sequentially by selecting possible solutions and testing these solutions by guess and check to see if they satisfied the conditions as stated in the problem. When a possible solution did not satisfy all conditions, it was discarded. Finally, one solution was arrived at.

The student gave a full and complete explanation of why particular numbers were chosen. The computation supported the explanation and the solution was clearly stated.

Circle **one** of the following words you would use to describe this problem:

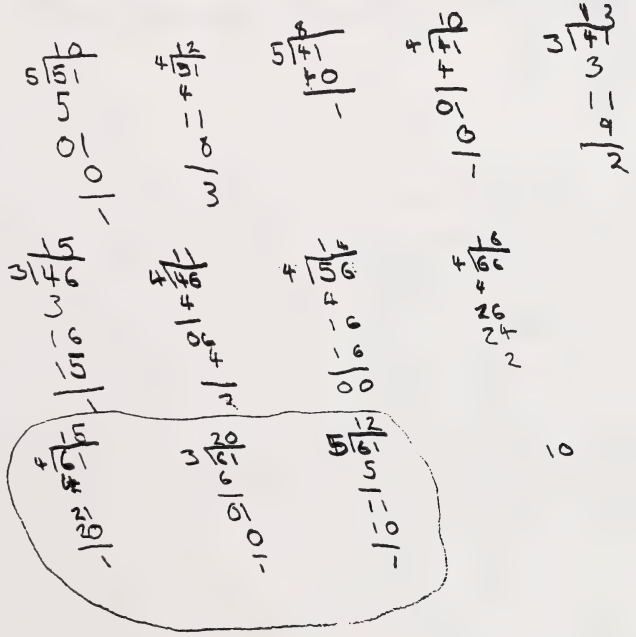
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Martin counted the marbles he had collected.
He counted more than 40 but less than 70.
When he put the marbles in groups of 5 he had 1 left over.
When he put them in groups of 4 he had 1 left over.
When he put them in groups of 3 he had 1 left over.
How many marbles did Martin collect?

SHOW AND EXPLAIN ALL YOUR WORK



I started to do the 51 but the 4 didn't work. the reason I thought of 51 was because I knew the that 5 would work. So I tryed 41. that didn't work. Then 46, that didn't work. so after I did all the things on the left I finaly came to an answer.

▲ *Partial* ▼

The student tried possible solutions of multiples of 5 plus 1, then tested by guess and check to see if a remainder of 1 would occur when dividing by 3, 4 and 5. The solution was circled.

The student understood the problem and began in a manner that led to a solution but failed to state the final solution. It might be recommended that a further interview or conversation with the student be held to affirm the student's understanding.

Circle **one** of the following words you would use to describe this problem:

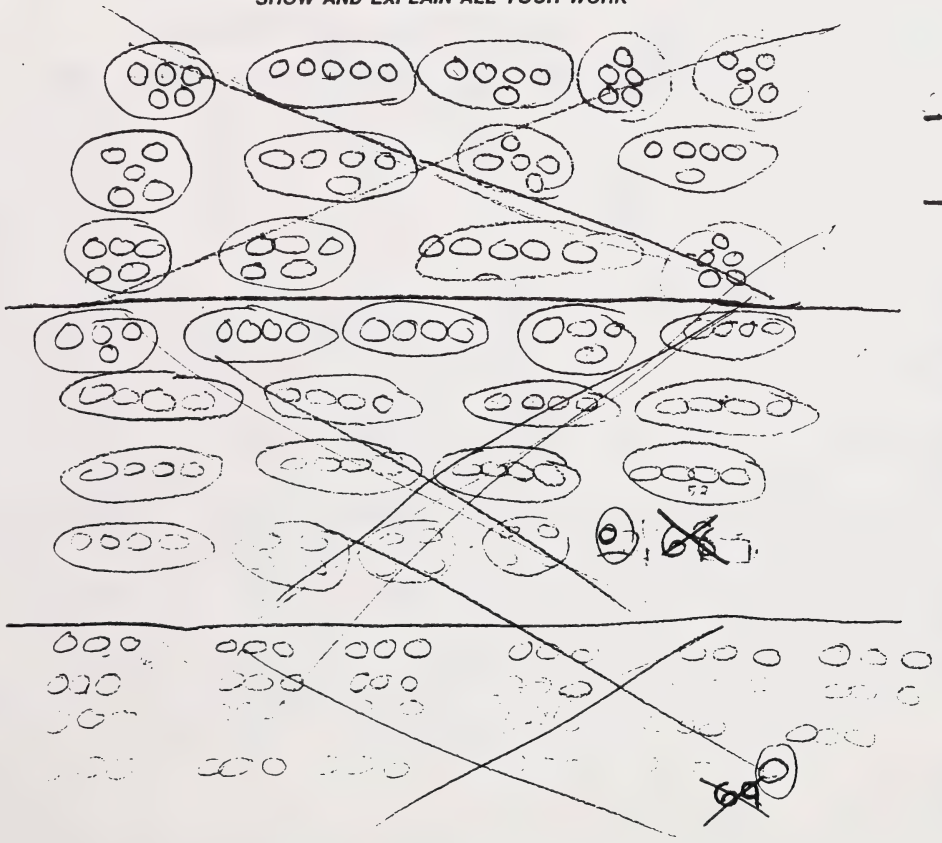
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Martin counted the marbles he had collected.
He counted more than 40 but less than 70.
When he put the marbles in groups of 5 he had 1 left over.
When he put them in groups of 4 he had 1 left over.
When he put them in groups of 3 he had 1 left over.
How many marbles did Martin collect?

SHOW AND EXPLAIN ALL YOUR WORK



40 + 5 + 3 = 52

40 + 5 + 3 = 48

40 + 4 + 3 = 47

40 + 4 + 5 = 49

70 - 5 - 3 = 58

70 - 5 - 3 = 62

70 - 4 - 3 = 66

70 - 4 - 5 = 61

first I read the problem

▲*Preliminary*▼

The student began by illustrating multiples of 3, 4, and 5 which led nowhere. The student changed tactics and began to add different combinations of 3, 4, and 5 to 40 and to subtract similar combinations from 70.

The student gave no explanation of why the problem was approached in this manner. The student seemed not to understand the problem and it might be beneficial to discuss the solution with the student to discover why there was no follow through to the initial approach.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

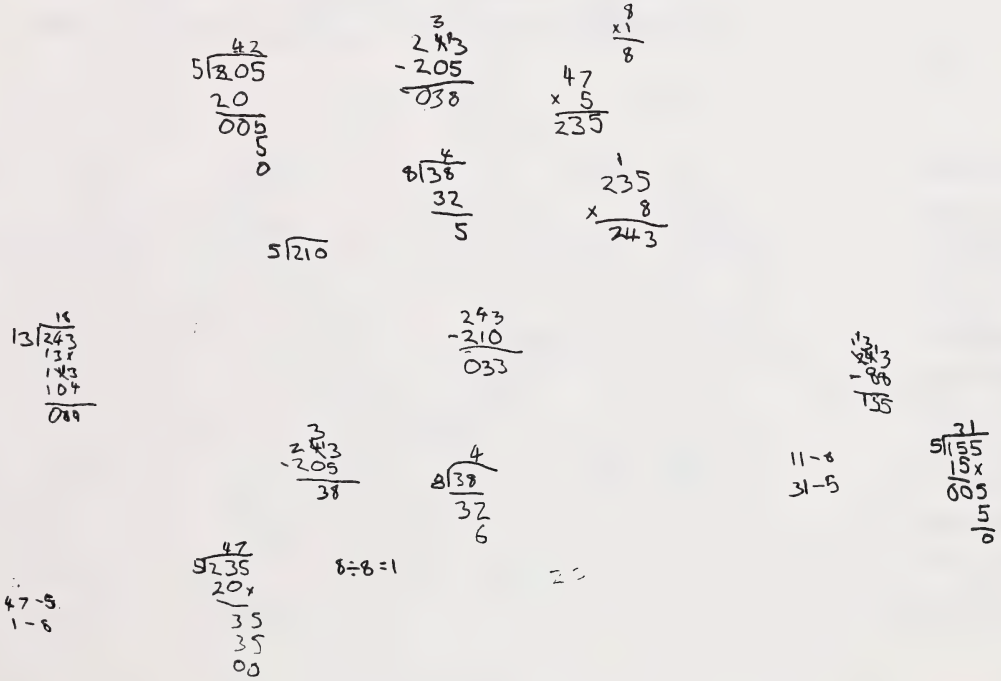
Discouraged *Satisfied* *Frustrated* *Happy*

How many five-cent and eight-cent stamps can you buy for exactly $2.43?

Are there other solutions?

SHOW AND EXPLAIN ALL YOUR WORK

One of the aswrs 47-5 cent stamps and 1-8 cent stamp. The second answer was 31-5 cent stamps and 11-8 cent stamps.



Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲*Complete*▼

This student used an interesting approach that began with the premise that 8 cents and 5 cents equals 13 cents. S/he then divided $2.43 by 13 to see if a possible solution could be found. This was not a possible solution. Next, random numbers larger than 200 and multiples of 5 were chosen and tested with a multiple of 8. The first solution was found. The second solution followed the statement "Are there other solutions?" S/he tried 11 eight cent stamps and subtracted this amount from $2.43. The result was divided by 5 and found there was no remainder.

The student explanation is inferred by accompanying computation. The teacher has to assume some choice of solutions is done mentally before arriving at possible solutions. An interview with the student would give insight into his path of logic.

How many five-cent and eight-cent stamps can you buy for exactly $2.43?

SHOW AND EXPLAIN ALL YOUR WORK

We know

only can spend $2.43 to buy 5¢ and 8¢ stamps

question

how many stamps can we buy

Plan

$2 \times 20 = 40$

5, 10, 15, 20, 25, 30, 35

$47 + 8 = 55$

Answer

I can buy 55 stamps with $2.43

Are there other solutions?

Yes

$2 \times 20 = 40$ + 5 8¢ stamps = $2.40

answer → this is not right because there is 3¢ left

$1.60 + 40¢ = $2.00

25 8¢ stamps = $2.00

2.00, 2.05, 2.10, 2.15, 2.20, 2.25, 2.30, 2.35

7 5¢ stamps

$2.35 + 8¢ = $2.43

Answer: I can buy 33 stamps with $2.43

▲*Partial*▼

The student tried several approaches but never followed through on these possibilities. All approaches could have led to a correct answer.

The explanation was incomplete and the solutions were not clearly supported. The student understood the problem but the answers are incorrect. The answers were stated as the total number of stamps rather than the number of each denomination.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

How many five-cent and eight-cent stamps can you buy for exactly $2.43?

SHOW AND EXPLAIN ALL YOUR WORK

We know

We have to count how many five-cent and eight-cent stamps can we buy for $2.43.

Plan

2 5
x40 10
00 15
80 20
25
30
35

Check 47
+ 8
55

Answer

There are 55 stamps.

Are there other solutions?

Yes

Plan

5 stamps in 25¢ | 20 stamps in $1.00 | 40 stamps in $2.00 | $2.00 + 43 = $2.43

Answer

There are 83 stamps.

▲ *Preliminary* ▼

The student began by finding values of 5 cent stamps but later misrepresented the information by combining the number of stamps with dollar values to arrive at a solution. Also, in the solution, the total number of stamps was given rather than the number of each denomination.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

There are fewer than 6 dozen eggs in a basket.
If I count them 2 at a time, there is 1 left over.
If I count them 3 at a time, there are none left over.
If I count them 4, 5, or 6 at a time, there are 3 left over.
How many eggs are there?

SHOW AND EXPLAIN ALL YOUR WORK

multiples of 3: 3, ~~6~~, 9, ~~12~~, 15, ~~18~~, 21, ~~24~~, 27, ~~30~~, 33, ~~36~~, 39, ~~42~~, 45, ~~48~~, 51, ~~54~~, 57, ~~60~~, 63, ~~66~~, 69, ~~72~~,

#'s Left: ~~3~~, ~~6~~, 9, 15, ~~18~~, 21, 27, ~~33~~, 39, ~~45~~, 51, ~~57~~, 63, ~~69~~

#'s 4 could go into leaving 3
4 = ~~15~~, ~~27~~, ~~39~~, ~~51~~, 63

#'s 5 could go into leaving 3
5 = 63

#'s 6 could go into leaving 3
6 = 63

Answer - (63)

- I found that the answer had to be less than 6 dozen (72)
- It had to be a multiple of 3, because it said "if 4 would, then 3 and in other, there are more left over." So I listed the multiples of 3
- Out of the numbers left, I took the numbers that four would go into, having 3 left over.
- Out of those, I took out the numbers that 5 would go into, having 3 left over.
- Since there was only one number left, I checked to see if 6 would go into it leaving 3.

- It worked
- I double checked and went through the procedure again
- It worked again.
- So I know that 63 is the right answer

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲*Complete*▼

This is a complete response. Before beginning the strategic process, it is recommended that students restate in their own words to demonstrate an understanding.

The student could have explained one more step - why the numbers 6, 12, 18, 24, etc.) multiples of 2 were crossed out. Also the student could explain in the "look back" why the even multiples of 3 had not been eliminated.

Organization, motivation, and confidence in approach were demonstrated.

The student's attitudinal dispositions reflect a positive response - fun and happy.

There are fewer than 6 dozen eggs in a basket.
If I count them 2 at a time, there is 1 left over.
If I count them 3 at a time, there are none left over.
If I count them 4, 5, or 6 at a time, there are 3 left over.
How many eggs are there?

SHOW AND EXPLAIN ALL YOUR WORK

understand: How many eggs are there if there is fewer than 6 dozen eggs, and count them by 2, there is 1 left over, if I count it 3 at a time and there is none left over, and if I count 4, 5 or 6 at a time, there is 3 left over.

Plan: make a picture or make a list

eggs	count by	amount counted	total	left over
~~71~~	2	35	70	1
~~71~~	3	23	69	2
~~69~~	3	23	70	0
~~69~~	2	35	70	1
~~69~~	4	17	68	3
~~69~~	5	14	70	1
~~67~~	2	35	70	1
~~67~~	3			
~~63~~	3	21	63	0
~~63~~	2	35	70	1
~~63~~	4	17	68	3
~~63~~	5			

} right

63	2	31	62	1 ✓
63	3	21	63	0 ✓
63	4	15	60	3 ✓
63	5	12	60	3 ✓

look back: There are 63 eggs. Yes th
is reasonable because it satisf- s
conditions

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

▲ *Partial* ▼

The student could demonstrate understanding better by using new words rather than just rewriting.

The approach is logical. It is recommended that in the process the student explain why even numbers were not included and 65 was eliminated.

This approach would require real perseverance and would prove to be tedious if the correct response had not been closer to 71 than to 1. In fact with the calculations the student has shown in the first part of the solution there is no clear indication of why 63 is the correct response in the student's mind. Later the student concludes that 63 is the correct response and with corrections in the calculations has proved to be so with the exception of completing the multiples of 6.
It is recommended that the student clearly justify the reasons for choosing 63 as the answer. The student's attitudinal response indicates satisfaction and fun.

There are fewer than 6 dozen eggs in a basket.
If I count them 2 at a time, there is 1 left over.
If I count them 3 at a time, there are none left over.
If I count them 4, 5, or 6 at a time, there are 3 left over.
How many eggs are there?

SHOW AND EXPLAIN ALL YOUR WORK

Problem: How many eggs are there?

12 (1 dozen)
x 6
72

The number has to be: less than 72
odd
a multiple of 3
3 more than a multiple of 4, 5 or 6

Odd multiples of 3 (not divisible by 4, 5, 6):
3, 9, 21, 27, 33, 39, 51, 57, 63, 69

3 less is 60.
60 is divisible by 4, 5, 6.
It is also a multiple of 3 and an odd number

a faster way would be to take the multiples of 6, and take the ones divisible by 4 & 5. Then I could add 3 to those and see which ones are divisible by 3, and odd.

The way I did it was: I took all the odd multiples of 3 that weren't divisible by 4, 5, or 6, and minused 3 from the ones 3 more than a multiple of 5, (ended in 3 or 8), and saw the one (after subtracted) that was a multiple of 4 and 6.

Another way would be to find the LCM of 4, 5 and 6, then see if it was a multiple of 3. That would also give me the answer.

ANSWER: 63
There are 63 eggs in the basket

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Elegant* ▼

The student displays a clear understanding of the approach to be taken. The student demonstrates some preliminary internal thinking whereby possibilities have been narrowed. The strategic approach is very sophisticated if we believe that a list has been compiled through purely internal thinking. It is recommended that this student explain the reasoning in the process of eliminating of all possibilities (3, 9, 21, 27, ...69) except for 63. It appears that a mental test of the multiples of 3 has taken place to see which number(s) satisfy all the conditions.

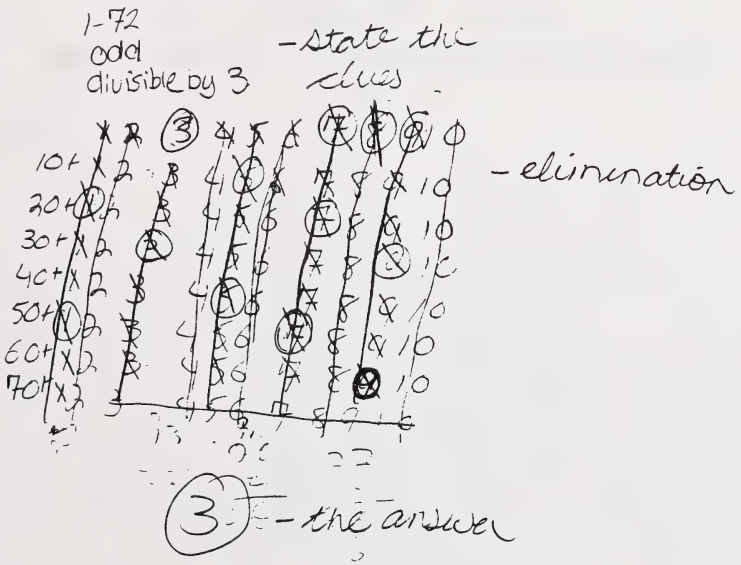
The student demonstrates creative thinking in realizing that the mathematical algorithm of finding LCM or of listing the multiples of 6 and using the process of elimination would have been a more direct approach.

The student's approach is evident because knowledge of all three approaches are valid. Reflection of the way I think it was... indicates a full and clear understanding of the problem. Attitudinal responses are positive, challenging, and happy.

There are fewer than 6 dozen eggs in a basket.
If I count them 2 at a time, there is 1 left over.
If I count them 3 at a time, there are none left over.
If I count them 4, 5, or 6 at a time, there are 3 left over.
How many eggs are there?

it seems like a difficult question, but a simple answer!

SHOW AND EXPLAIN ALL YOUR WORK



Explanation:
the answer is 3! there are 3 eggs in the basket. the answer is not always a hard number! the way this works is, 3 is less than 72, 3 has 3 left over for 4, 5, 6!

3 is 2 with one left over
3 groups of 4 with 3 left over
3 groups of 5 with 3 left over
3 groups of 6 with 3 left over

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲*Complete* ▼

This student has demonstrated a complete and unique response to the problem. The student has related some of the facts but should have included all of the conditions to demonstrate complete understanding. The strategic process of elimination requires further explanation and clarity.

It appears the student initially eliminated all even numbers, then circled all odd multiples of 3, making several errors in the process (3, 7, 8, 63). At this point, the student probably checked the remaining numbers realizing that none of these satisfied the last criteria. It is assumed that the student then began to check all numbers again, starting with 1 and found that 3 would indeed meet the conditions.

It is highly recommended that this student be interviewed and encouraged to verbalize the thinking processes.

This student shows perseverance and motivation. Confidence that the response is indeed correct is shown in the statement: "the answer is not always a hard number."

The student's attitudinal response, confusing and happy are indicators of the possible struggle experienced in completing this problem.

Lines

1
2
3
4
5

There are fewer than 6 dozen eggs in a basket.
If I count them 2 at a time, there is 1 left over.
If I count them 3 at a time, there are none left over.
If I count them 4, 5, or 6 at a time, there are 3 left over.
How many eggs are there?

SHOW AND EXPLAIN ALL YOUR WORK

How many eggs are there?

lines	What we found out
1st	< 6 dozen which is equal to 72
2nd	$2 \times n+1$ = number of eggs
3rd	eggs are multiple of 3
4th	4,5 or $6 \times n+3$ = number of eggs

Rules

1. There is less than 72 eggs (can be 1-71).
2. There are an odd number of eggs.
3. You can divide the number of eggs by 3 evenly, without any remainders.
4. If you subtract 3 from the number of eggs you will get a common multiple of 4,5 and 6

Eliminating

Has to be odd numbers under 72
(1,3,5,7,9 ... 71)
Highest number it could be for rule #3 is 69 because that is highest multiple of 3 that is under 72.
multiple of 3 ≠ 71
" " ≠ 70
" " = 69
Lowest number it could be for rule #3 is 3 because that is the lowest multiple of 3.

Rule #4

$2\overline{|4} = 2^2$ $5 = 5$ $2\overline{|6} = 2 \times 3$

LCM = 60

$4 = 2^2$
$5 =$
$6 = 2 \times 3$

$2^2 \mid 3 \mid 5$

Highest number it could be for rule #4 is 63 because LCM of 4,5,6 is 60 and 60 plus 3 is 63. Lowest number it could be is also 63.

Summary

63 is less than 6 dozen (72)
63 is equal to $n \times 2 + 1 = 63$
63 is a multiple of 3 ($3 \times 21 = 63$)
63 is equal to $n \times 4, 5 \text{ or } 6 + 3$

$15 \times 4 + 3 = 63$
$12 \times 5 + 3 = 63$
$10 \times 6 + 3 = 63$

Conclusion

There are 63 eggs in the basket.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

▲ *Elegant* ▼

The student displays a mature approach to solving the problem by restating the conditions to be met in new words in two different methods. The student is creative in demonstrating two strategic processes - (1) Elimination and (2) a more logical method of selecting the LCM of 2, 3, 4, 5, and 6.

The student's summary is a mature reflection of thought processes, justifying the answer, thereby demonstrating understanding of the question.

The student's attitude and response demonstrates confidence and satisfaction in organization and solution. The student appears highly motivated and has a logical approach to this problem.

Two men and two women teach at Middle Trail School.
Their names are Angela, Dorothy, Jake and William.
One teaches art, one teaches English, one teaches math,
and one teaches science.
From the following clues, name the subject each person teaches:

1. Dorothy's brother teaches science and
   is married to the English teacher.
2. The art teacher is an only child.
3. Both William and the math teacher are single.

SHOW AND EXPLAIN ALL YOUR WORK

Problem: Who teaches what subject?

Plan: Chart

Do:

X = not possible O = possible

Subjects	Angela	Dorothy	Jake	William
Art	X	X	X	O
English	O	X	X	X
Math	X	O	X	X
Science	X	X	O	X

- since Dorothy's brother is married to the English teacher, a male teaches science and a female teaches English.
- the art teacher cannot be Dorothy because he/she is an only child and Dorothy has a brother.
- william is single so he can't be Dorothy's brother and he can't teach science.
- William teaches art by elimination
- Jake is Dorothy's brother and teaches science by elimination.
- Jake has to be married to Angela so she is the English teacher
- that leaves Dorothy as the Math teacher.

Conclusion

Angela teaches English
Dorothy teaches Math
Jake teaches Science
William teaches Art

▲*Complete* ▼

The student has read and clearly understood the clues, made some valid observations and conclusions. The work shown here exemplifies a student with confidence in his logical reasoning, one conclusion seems to flow naturally from the one previous to lead to the answer.

This student is able to communicate very clearly and concisely leaving no room for error or uncertainty. The student indicated that the problem was fun to do and was happy after doing it, identifing a self-motivated, confident problem solver possessing good strategic approaches and processes.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Two men and two women teach at Middle Trail School.
Their names are Angela, Dorothy, Jake and William.
One teaches art, one teaches English, one teaches math,
and one teaches science.
From the following clues, name the subject each person teaches:

1. Dorothy's brother teaches science and
   is married to the English teacher.
2. The art teacher is an only child.
3. Both William and the math teacher are single.

Subject	Clues	Answer
Art	not Dorothy	William
English	not Dorothy, a woman	Angela
Math	not William	Dorothy.
Science	not Dorothy, a man.	Jake.

SHOW AND EXPLAIN ALL YOUR WORK

Problem: Who teaches each subject?
Plan: Elimination from the clues, make a chart

Clues: 1. Dorothy's brother teaches science & is married to the English teacher.
So we know that Dorothy doesn't teach science or English, a man teaches science and a woman teaches English - Angela teaches English.

2. The Art teacher is an only child.
So we know Dorothy, Angela don't teach art, because Dorothy has a brother, Angela teaches English, so it's a man

3. Both William & the math teacher are single
So we know William isn't the math teacher - it must be Dorothy

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

Even though the student has the correct answer, there is no explanation why William and Jake teach the subjects they do. The reasoning from clue 1 and clue 2 is complete and clear and indicates that the student fully understands the clues and is able to think and reason well logically. It appears that this student, after some valid observations and finding an answer, forgot or found it unnecessary to explain why that conclusion was indeed correct. The chart the student made also indicates this - for the clues for each subject, there are only one or two eliminations, but this is not continued once the correct person for each teaching position is discovered.

Perhaps, if this student were to be made aware of this shortcoming in problem solving, more perseverence and fuller explanations will occur in future problem solving. Giving the correct solution is only one part of problem solving.

The student's response does indicate that he has a strategic approach and process which should be followed through with justifications. The response that it was fun to do and happy feelings indicates a motivated and confident problem solver who prefers to discuss the reasoning of a problem verbally in class rather than having to write it all down.

Two men and two women teach at Middle Trail School.
Their names are Angela, Dorothy, Jake and William.
One teaches art, one teaches English, one teaches math,
and one teaches science.
From the following clues, name the subject each person teaches:

1. Dorothy's brother teaches science and is married to the English teacher.
2. The art teacher is an only child.
3. Both William and the math teacher are single.

SHOW AND EXPLAIN ALL YOUR WORK

Problem: Who teaches which subject?

Plan: Chart

Do:

Subject	Angela	Dorothy	Jake	William
Science	♀	X	///	♀
Math	♀	□	///	☆
English	○	X	♀	△
art	///	♀	///	///

PEOPLE

FACTS FOR CHART

X○☆□△

mean can't be possible

♀ = the profession any person are correct

- since Dorothy's brother teaches science, and her husband teaches English - she can't teach science or English = X's
- English teacher is a man, because he is married to a woman = so Angela can't be the english teacher
- William is single, so he can't be the science teacher - = ☆
- since the math teacher is single - Dorothy is married, so she can't be the math teacher - □
- since Dorothy teaches art, no one else can teach art - ///
- science teacher is a man - Dorothy's brother, so Angela can't teach science - ○

- math was the only profession left under (for) Angela - so she is the math teacher = ♀
- Art is the only profession left under Dorothy - so she is the art teacher = ♀
- William is single, so he isn't married to Dorothy - so he can't be the English teacher = △
- the only profession left under William is science - so he is the science teacher = ♀
- the only profession left open, is English, so Jake is the English teacher = ♀
- /// = know not possible from other clues

- I thought that Dorothy was married to the English teacher, and her brother teaches science.

Conclusion: Angela teaches math.
Art is taught by Dorothy.
Jake teaches English.
William teaches Science.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* (*Happy*)

▲ *Preliminary* ▼

This is an example of a student's response in the preliminary category. It appears this student may be hard-working, persevering and perhaps confident.

In the student's approach, it is unfortunate that there is one incorrect conclusion at the beginning. It is recommended that this student be interviewed and be allowed to explain the thinking. Perhaps, without too much prompting, this student might be able to realize the error and the conclusions would be changed.

This student appears to be confident. A second chance - to re-evaluate the conclusions - would appear to be necessary and fair.

The student's attitudinal response appears to indicate the belief that the answer is correct without question. It may be an idea to have the student reason the problem verbally to the teacher, in hopes of explaining the errors in the initial reasoning. There is an example of where a student should "look back" and reflect more frequently and thereby improve.

Grade 8

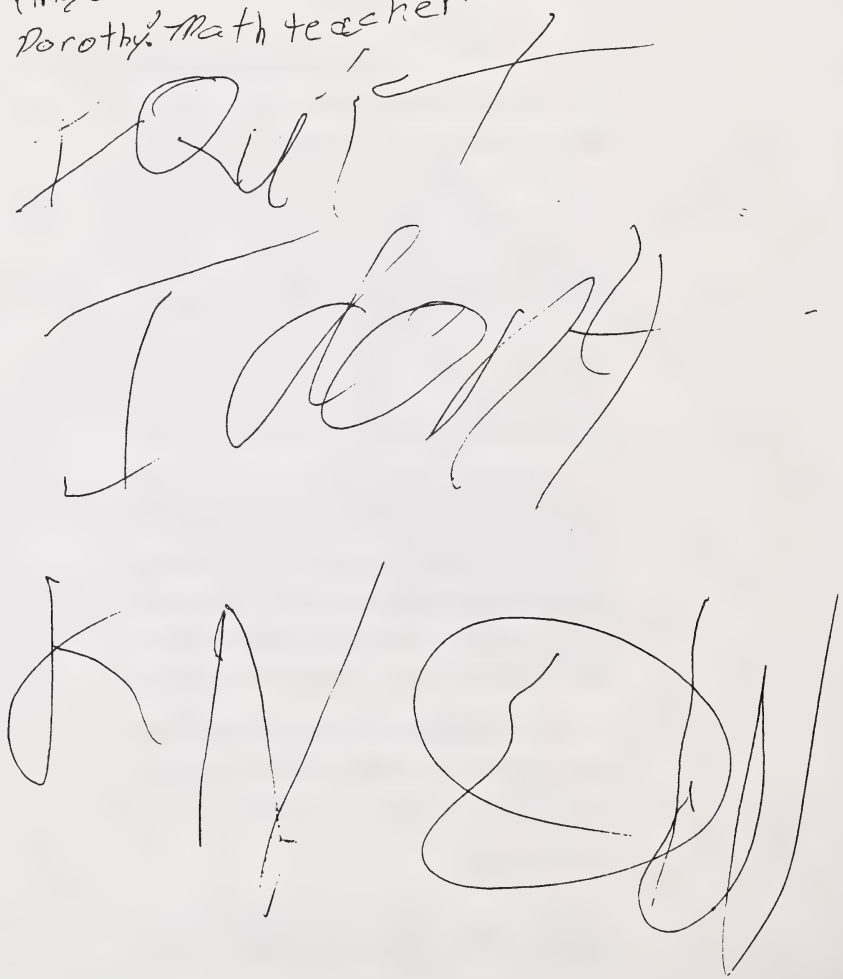
Two men and two women teach at Middle Trail School.
Their names are ~~Angela~~, Dorothy, Jake and William.
~~One teaches art, one teaches English, one teaches math,~~
and one teaches science.
From the following clues, name the subject each person teaches:

1. Dorothy's brother teaches science and is married to the English teacher.
2. The art teacher is an only child.
3. Both William and the math teacher are single.

SHOW AND EXPLAIN ALL YOUR WORK

Understand: 2 men 2 women teach
There names are Jake William
Angela Dorothy one teaches art,
one English one science, one math,
1. Dorothese brother teaches science
2. The are teacher Sanandy Child
~~3~~ both William and the Math teacher are single
Plan Check Mental Work

Do
~~William's~~
Angela teaches Art
Dorothy Math teacher



Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Preliminary* ▼

This is an example of a preliminary response written by a frustrated student. It exemplifies the confusion in the student's mind. It seems the student finds it difficult to assimilate so much information.

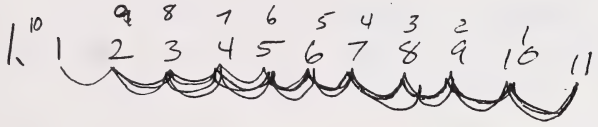
It is recommended that a student of this nature be allowed to experience some success in solving logic problems beginning with manipulatives, i.e., arranging five people according to height given a set of clues and attribute blocks or counters.

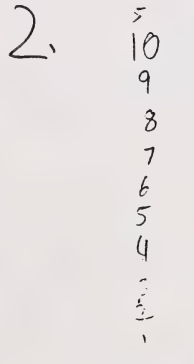
A student having these difficulties would benefit best from working in a one-to-one situation in order to develop some positive dispositions and stimulate quality outcomes. This student needs to verbalize the meaning of each clue and justify the conclusions.

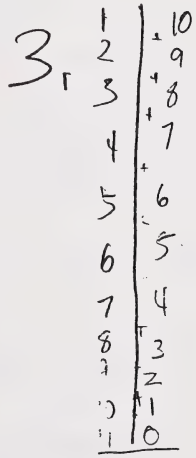
This student's attitudinal response was one of confusion and frustration.

Eleven people are introduced to each other,
and each person shakes hands with each other exactly once.
How many handshakes are exchanged altogether?

SHOW AND EXPLAIN ALL YOUR WORK

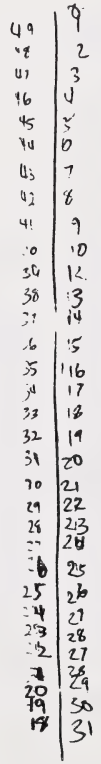
1. 

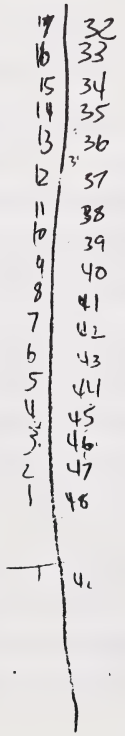
2. 

3. 

55 handshakes were exchanged

Explain how you would solve the problem
if 49 people were to shake hands once with each other.





I would use a chart similar to the first problem as shown here. I would not use a drawing method because it would get too complicated. By puting down all the numbers up to 49 and using common sense. If the first person ~~shook~~ 48 handshakes then made #2 would make 47 handsha because ~~she~~ #1 already shook with #2, and so on.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

The student developed a strategic approach by using a diagram, listing totals and finally writing these in somewhat of a chart form, although headings and explanation are incomplete.

The first page should not be considered as three separate approaches to the problem as the student has labelled, but rather three steps in one approach.

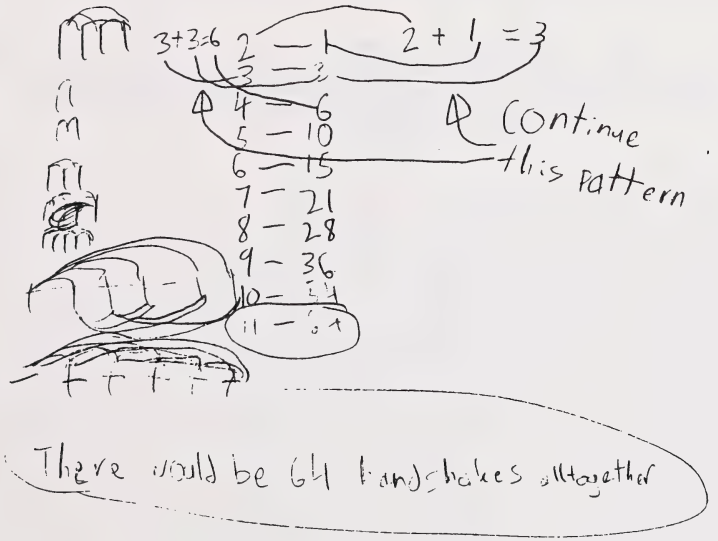
The student realized the diagram approach would be too complicated to continue. This strategic process involves "common sense." In an interview, it would be helpful if the student would develop any logical pattern from his table, i.e.: *1, 48*
2, 47 — the sum being 49
3, 46

Had the student explained what to do with those numbers (i.e., add) without necessarily finding the sum, this solution could have been considered complete. Could the student then realize that rather than listing all the 1-49 people and 0-48 handshakes, there is a simpler method.

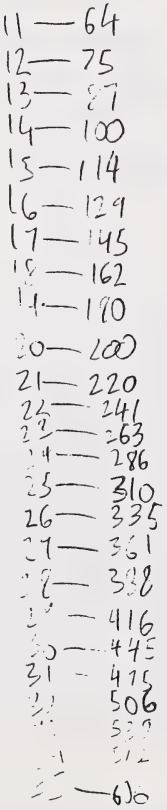
There has been no evidence that the student used the sum of 55 from page one to apply to the extension part of the problem.

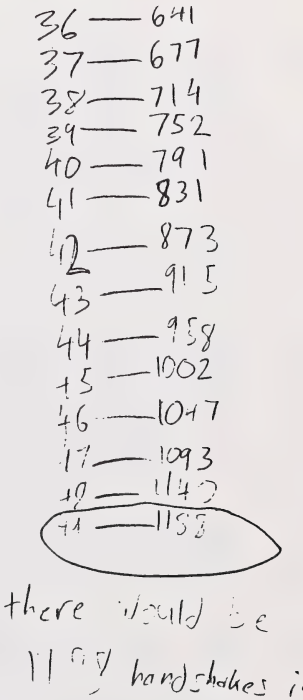
Eleven people are introduced to each other,
and each person shakes hands with each other exactly once.
How many handshakes are exchanged altogether?

SHOW AND EXPLAIN ALL YOUR WORK



Explain how you would solve the problem
if 49 people were to shake hands once with each other.





Circle **one** of the following words you would use to describe this problem:

Long – *Boring* *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy* that I'm done

▲ *Partial* ▼

The student has approached the problem initially using diagrams and seems fairly confident after doing 6 handshakes to attempt to explain the pattern which was developed. Unfortunately, the arithmetic at the 10th person is incorrect resulting in the solution 64 rather than 55. This written explanation of thought processes is incomplete although the use of arrows does provide some evidence of the understanding.

The student's arithmetic approach would provide an accurate sum (albeit boring) if not for the initial incorrect sum of 11 people = 64 handshakes and 3 incorrect sums arrived at in the process. S/he is certainly motivated enough to complete the process and should be complemented for the efforts.

No reflection has been provided.
The student's attitudinal response is interesting. Unfortunately, the method of solving the problem is "long and boring" and he is "happy that I'm done."

Eleven people are introduced to each other,
and each person shakes hands with each other exactly once.
How many handshakes are exchanged altogether?

SHOW AND EXPLAIN ALL YOUR WORK

11 - 10
10 - 9
9 - 8
8 - 7
7 - 6
6 - 5
5 - 4
4 - 3
3 - 2
2 - 1
1 - 0

55 handshakes

Everytime one person
shakes hands; one
less handshake is needed

Explain how you would solve the problem
if 49 people were to shake hands once with each other.

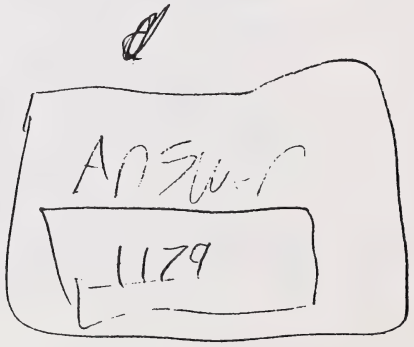
48 47 46 45 44 43 42 41 40 39 38 37 36 35 34 33 32 31

671
400
1081
+ 48
1129
+50
1179

30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11

410

I noticed a pattern. every time
the person shook hands
one less handshake was needed
The first result was to 49
to people -1 + everyone else.
(n-1 + n)x2 I added every =
between 11 and 48 + 1 and 10.
~~I got lost up~~ Then I added
48 because 49-1=48. The
answer was 1179



Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

This response would be rated partial. It would be helpful if the student had explained the concluding statement - "every time one person shakes everyone's hands, one less is needed." A complete understanding isnot shown.

In the extension, the student's approach is logical although the arithmetic is inaccurate. (Sum should be 711 rather than 671.)

It would be helpful to know several reasons for the student's thinking:

1. *Why did s/he add 671 (sic 711) + 410 + 48 +50? There seems to be no reason for this. (711 + 410 + 55 = 1176)*
2. *What is the student's thinking in his statement (n − 1 + n x)?*

Is his solution based on memory of completing a similar problem previously?

The student appears satisfied with the response although no reflection is completed.

The attitudinal response is "boring."

Eleven people are introduced to each other,
and each person shakes hands with each other exactly once.
How many handshakes are exchanged altogether?

SHOW AND EXPLAIN ALL YOUR WORK

Problem: How many handsakes for 11 people

Procedures: Diagram
* Charts *
Powers

# of people	# of handshakes
11	10
10	9
9	8
8	7
7	6
6	5
5	4
4	3
3	2
2	1

= 55 handshakes

Explanation: In the chart, the 1st two rows that I filled out, I used diagrams to help me. Then I saw a pattern. I saw that the # of handshakes were one less than the # of people, so I filled out the rest of the chart without using diagrams. Then I added the # of handshakes together to get 55 handshakes

Explain how you would solve the problem
if 49 people were to shake hands once with each other.

49 39
48 38
47 37
46 36
45 35
44 34
43 33
42 32
41 31
+40 +30
445 345

2
445
345
245
145
+ 45
1225

Answer: 1225

I added the 40's together getting 445, then the 30's together getting 345, so the 20's must get 245. I added all these together getting 1225

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Complete* ▼

The student has rephrased the problem and has selected two appropriate procedures to arrive at a solution.

It would be helpful to know why the student did not include the series of diagrams.

The student established a pattern and completed the chart, demonstrating confidence in the strategic approach. The student did not complete a reflection (look-back) of the solution.

*In the extension, the student's creativity** is demonstrated in the development of a pattern adding the 40s, the 30s, etc.*

Questions by the interviewer of the student might be:

1. *When there were 11 people, the number of handshakes was the sum of the number 1 to 10 making 55. How can you use this information in the extension?*

i.e.:
$$1 + 2 + \ldots 10 = 55$$
$$11 + 12 + \ldots 20 = 155$$
$$21 + 22 + \ldots 30 = 255$$
$$\_31 + 32 + \ldots 40 =$$
$$\_41 + 42 + \ldots 50 =$$
$$\text{Total} = \_50\ \_49 = 1176$$

OR You have established a pattern which seems to make sense. Check what you have.

The student has established a pattern not related to the sum of 55 on page 1. This pattern would have worked; however, s/he simply forgot to eliminate the #49 at the end.

2. *You have established a pattern which seems to make sense. Looking back, explain to me why you started with the 11th person had 10 handshakes.*

   Now look at the completion of your new pattern. What numbers did you add?

   ***Hint:** The 49th person would shake hands with how many people?*

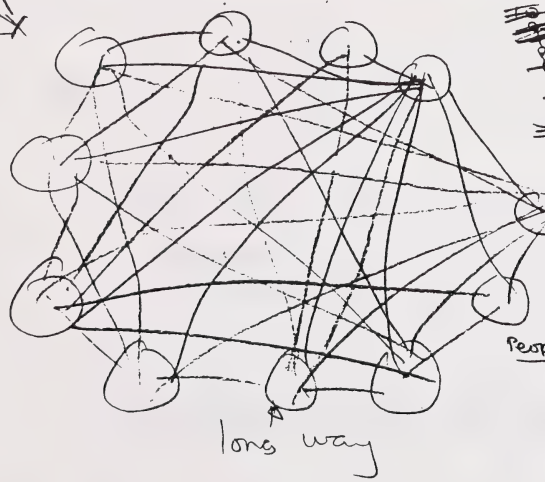
   How can you change this pattern?

Grade 8

3-3
4-8 2+3 21
10
15

Eleven people are introduced to each other,
and each person shakes hands with each other exactly once.
How many handshakes are exchanged altogether?

SHOW AND EXPLAIN ALL YOUR WORK



long way

3 short way

55 people

People	shakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21

all you need is to find out the diagonals of the polygon the people formed, then add the sides

~~pento~~ a pyrimid

-21
8 + 28
9 - 36
10 - 45
11 - 55

x + x → handshake
eg 8 + 28 = 36

Explain how you would solve the problem
if 49 people were to shake hands once with each other.

I would add the numbers from 49 to 2 together to get the answer

49
- (49-2) add
2

49
48 2
47 3
46 4
45 5
44 6
43 7
42 8
41 9
40 10

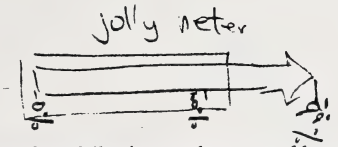
39 11
38 12
37 13
36 14
35 15
34 16
33 17
32 18
31 19
30 20

23
x50
1150 +
29 21
28 22
27 25
26 24
25

49
+25
74
+1150
1224

1224

* This is the third time I've seen this question, please make out different question



Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy* *jolly*

▲*Complete* ▼

This response is one ranging between completeness and elegance.

The student demonstrates complete confidence in the response by writing three approaches to solving the initial problem, the long way, the short way and by pattern. Note the simpler diagram above (4 people) and the establishing of a pattern.

The student has demonstrated a strategic process and appears motivated sufficiently to design three approaches to find the solution. Unfortunately, the student was so confident in the solution that the student did not return to correct the sums (page 2). (It is not 49 + 2 but 48 + 1, 47 + 2, etc., that make the total.)

It would be helpful to have an interview with this student. Why begin the short way with 11 rather than 10? Was 66 the solution initially, later changed to 55? Was this because of a remembered solution from a previous time? Why did she mention "pasco's pyramid? (sic) and how did this relate to the solution? In the extension, why begin with 49 adding ... to 2 when 11 was eliminated in her original sum (previous page).

How much, however, has seeing the problem for the third time affected the confidence, creativity and finally the solution. Interestingly, the student's attitude response was "boring."

Bob, Barbara and Bill have different schedules for visiting the library. Bob goes every third day, Barbara goes every fourth day and Bill goes every fifth day. The last time they were at the library together was on a Tuesday. In how many days will they again be at the library and what day of the week will it be? (The library is open seven days a week.)

SHOW AND EXPLAIN ALL YOUR WORK

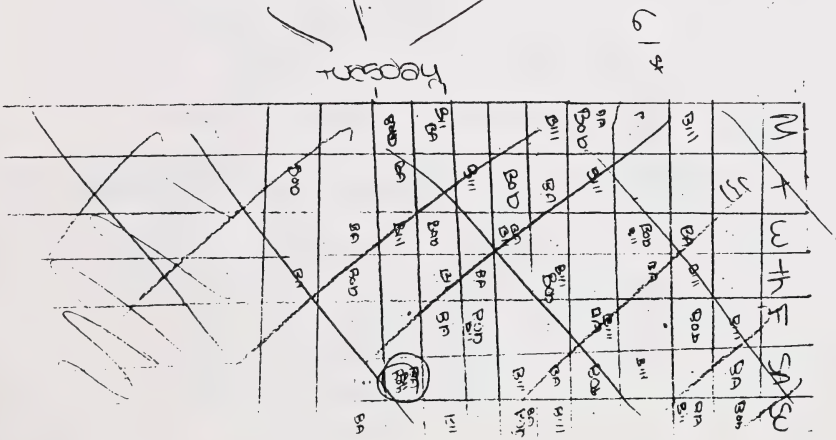
understood- how many days will they again be at the library and what day of the week will it be?

plan- draw a diagram

do- Bob, Barbara, Bill

5th day 4th day 3rd day

Tuesday



61st

Mon	tu	we	th	Fr	SA	Sun	S
				Bill	BA	Bob	
Bill		BA	Bill	Bob		Bill BA	
		Bob Bill	BA		Bill		
Bob BA	Bill			Bill BA	Bob		
Bill	BA		Bob Bill		BA	Bill	
	Bob	BA Bill			Bill	Bob BA	
	Bill		BA	Bill Bob			
BA		Bob	Bill	BA		Bill	
Bob	BA	Bill			Bob BA		
		BA	Bob			BA	

61st

look back - Bob, Barbara, and Bill all land on the same day on the 61st (counting Tuesday).

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Complete* ▼

Even though this student's work is being classified as complete it would have been enhanced with a full explanation in looking back. The work here would probably exemplify a student who is a "plodder" who has to see all the process step by step even though it becomes tedious and before one can draw valid conclusions. It seems that, because the student showed a chart with every day for each person visiting the library on it, it was not deemed necessary to further justify the answer. This particular student probably is motivated and hard-working but lacks in confidence and creativity which is reinforced by the student's response that the problem was challenging and that s/he was happy upon completion.

A student of this nature would probably have difficulty in generalizing a rule or skipping any steps in procedures and would benefit from problem solving experiences where the nature of the problem does not lend itself to the possibility of a chart or similar to display each clue or fact. This student would probably also benefit from working cooperatively with others sharing ideas and procedures.

Bob, Barbara and Bill have different schedules for visiting the library. Bob goes every third day, Barbara goes every fourth day and Bill goes every fifth day. The last time they were at the library together was on a Tuesday. In how many days will they again be at the library and what day of the week will it be? (The library is open seven days a week.)

SHOW AND EXPLAIN ALL YOUR WORK

Problem: What day of the week will it be and in how many days will Bob, Bill and Barbara be at the library together?

Plan: diagram, find LCM

Do: LCM of -3 4 5 (3=third day 4=fourth day 5=fifth day)

-3 and 4 don't go into 5

" " 10

" " 15

" 20

" 25

" 30

" 35

" 40

" 45

" 50

" 55

yes! 3, 4, 5 go into (60) 60

I am doing multiples of 5 because 5 is the biggest number.

3x4x5=60

LCM of 3, 4, 5 = 60

So all of the children will meet in the library in 60 days

now, onto the day of the week → turn over

since they all met on Tuesday I should start counting from there

-I will divide 7 into 60, because there is 7 days in a week, and in 60 days the children will meet

$$7\overline{)60}\ 8,\ 56,\ 4$$

-about 8 weeks (56 days) and 4 days = 60

-so 8 Tuesdays later (8 weeks) +4 days which is Saturday

*I'm doing a calendar so I know what day it will be 4 days later

Mon	Tues	Wed	Thurs	Fri	Sat	Sun
	X	1	2	3	(4)	

Conclusion: It will be Saturday, and 60 days later when all three children meet at the Library together.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲*Complete*▼

The student displays a strategic approach and process flowing logically with full explanations of each step and procedure. It is clear that the student fully understands the problem and what should be done to reach a valid conclusion. The student's work displays that confidence and security in the thinking and reasoning and probably exemplifies a student who is motivated and unafraid to tackle any or most problem solving situations. The student probably enjoys challenges and delights in solving difficult problems requiring a lot of reasoning. It can be noted that, here is an example of a student who has no difficulty in communicating in written form rather than just verbally.

The student responded that the problem was fun and s/he was happy reinforcing the above positive dispositions.

Bob, Barbara and Bill have different schedules for visiting the library. Bob goes every third day, Barbara goes every fourth day and Bill goes every fifth day. The last time they were at the library together was on a Tuesday. In how many days will they again be at the library and what day of the week will it be? (The library is open seven days a week.)

SHOW AND EXPLAIN ALL YOUR WORK

Problem: What is the LCM of 3, 4 & 5? (When will they be at the library together?)

Plan: Find the LCM (& count how many days of the week that'll be).

$3\overline{)3}\ \ 1$ $\quad 2\overline{)4}\ \ 2\ \ 1$ $\quad 5\overline{)5}\ \ 1$

LCM: $3 = 1, 3$

$4 = 2^2$

$5 = 1, 5$

$= 1 \times 2^2 \times 3 \times 5$

$1 \times 4 \times 3 \times 5$

$4 \times 3 \times 5$

$12 \times 5 = 60$

In 60 days, they'll be at the library together again

$60 \div 7$ (one week) $= 8$ Remainder 4

In 8 weeks & 4 days

Thursday + 8 weeks = Thursday + 4 days is Monday!

Answer: In 60 days on a Monday they'll be at the library together.

▲ *Partial* ▼

This student displays a clear understanding of the problem situation and has good strategic approach and process but it is only a partial response because the student counted the four extra days from Thursday instead of Tuesday. This was probably an oversight in re-reading the problem. It is suggested that the student be encouraged to read the problem again and justify verbally why Monday would be the day. The student displays confidence and security in logical reasoning but needs to review the full look back aspect of problem solving.

Circle **one** of the following words you would use to describe this problem:

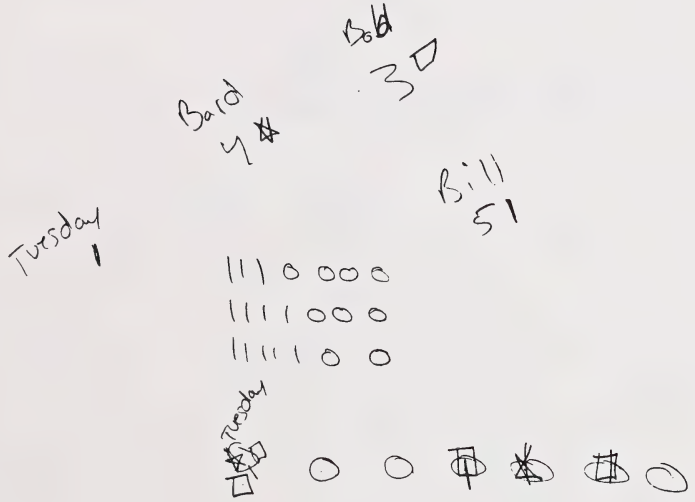
Boring *Challenging* *Confusing* *Fun*

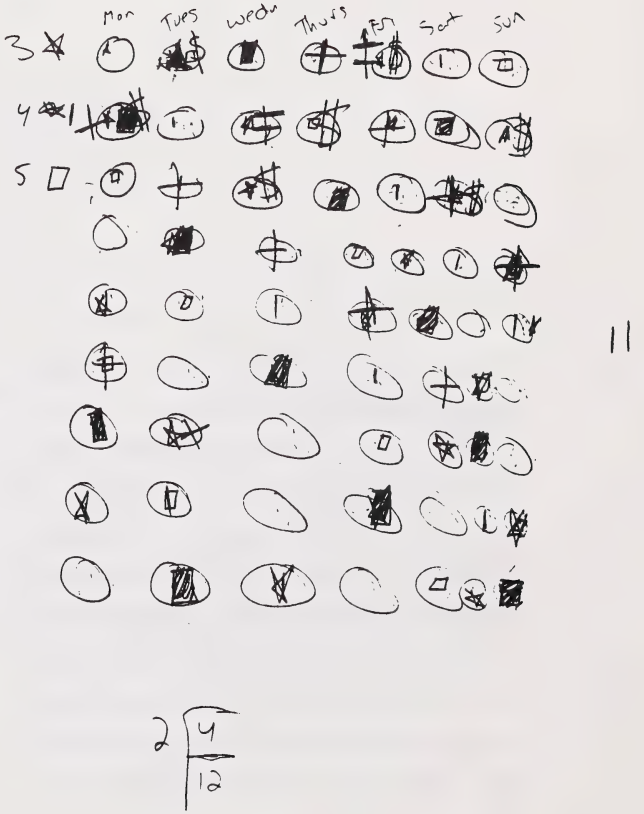
Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Bob, Barbara and Bill have different schedules for visiting the library. Bob goes every third day, Barbara goes every fourth day and Bill goes every fifth day. The last time they were at the library together was on a Tuesday. In how many days will they again be at the library and what day of the week will it be?
(The library is open seven days a week.)

SHOW AND EXPLAIN ALL YOUR WORK





Conclusion

They met on a Saturday after 60 days

Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

The work of this student exemplifies that of one who would prefer to verbalize responses and justifications rather than writing it on paper especially in sentence form. The response would indicate the work of a student classified as a "plodder," one who needs to show each clue or fact in order to be able to draw valid conclusions. The coded chart that the student drew here should lead to the correct response but the last Saturday - 60 days does not show all three symbols indicating it to be the day all three persons visit the library and yet that was the conclusion drawn.

Jennifer and Mandy like to ride bicycles.
Jennifer can ride 6 km in the same time Mandy can ride 4 km.
The girls want to ride 9 km to Fish Creek Park and arrive at the same time. How much of a head start should Mandy have?

SHOW AND EXPLAIN ALL YOUR WORK

(A) Jennifer could ride slower or -steroids

(B) Walk

(C) → GCF = 6 = 1 (2) 3 6 → 6 ÷ 2 = 3 → for every 3 km by Jenifer
4 = 1 (2) 4 4 ÷ 2 = 2 mandy rides 2

(Divide)

9 km ÷ 3 km = (3) km head start.

GFC

6 - 1 (2) 3 6 → Divide each by 2 (Jen) 6 ÷ 2 = 3 → So for each 3 km by Jenifer Mandy rides 2
4 - 1 (2) 4 (Man) 4 ÷ 2 = 2

So divide 9 by 3 which = 3

So Mandy gets a 3 km head start

▲*Partial* ▼

The student has shown some creativity in his listing of A, B, C. S/he understands how to use GCF but has not actually explained applying the concept of ratios. This explanation of 9 ÷ 3 = 3 km head start is incomplete. It would be helpful to ask the student to expain the thinking process.

This reflection is also incomplete as it is a restatement of what was done. It would be more appropriate to use a diagram to check on the answer.

Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Jennifer and Mandy like to ride bicycles.
Jennifer can ride 6 km in the same time Mandy can ride 4 km.
The girls want to ride 9 km to Fish Creek Park and arrive at the same time. How much of a head start should Mandy have?

SHOW AND EXPLAIN ALL YOUR WORK

Problem: How much of a head start should Mandy have?
Plan: equasion chart
Do: ratio = 6:4 (reduced) ratio = 3:2 (Jennifer to Mandy)
- For every 3 km that Jennifer bikes, Mandy bikes 2 km.
- So if the trip is 9 km, then Jennifer bikes 3 km, (3 x 3 km = 9 km) 3 times (that equals 9 km)
- When Mandy biked it, three times, her 4 km (in Jennifer's ride of 6 km) would be 6 km
- this new ratio is 9:6
- 9 (in the ratio) equals how far Jennifer can ride in the time that Mandy biked 6 km
- So, since Jennifer can bike faster than Mandy, you do: 9-6=□
- it equals 3: 9-6=3
- Mandy should get a 3km head start.

Mandy	Jennifer
4	6
8	12
12	18
16	24
20	30

CHART served no very big purpose.

← ratio still 2:3 (Mandy to Jennifer) (when numbers reduced)

(how many km biked compared to eachother)

Conclusion:
Mandy would need a 3 km head start.

Circle one of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

▲ *Partial/Complete* ▼

This student's response is rated as partial to complete. The student has restated the question and has suggested two approaches in the plan. However, a ratio approach is used which is not listed in the plan.

The explanation of strategic process is fairly well completed; there is confidence with the solution.

In the DO section, page 1, the explanation of the strategic process is fairly well completed. Only the line "When Mandy biked it three times, her 4 km (this 4 should read 2 km) would become 6 km." In other words, still following the 2:3 ratio.

The student's chart would be very valuable if there had been an extension section to this problem, i.e., how much of a head start does Mandy require if the girls are biking 30 km?

However, if the chart began appropriately with the reduced ratio of 2:3, the student would have included the ratio 6:9, and then the chart did in fact "serve a purpose"!

The student's attitudinal response suggests satisfaction with the work.

Jennifer and Mandy like to ride bicycles.
Jennifer can ride 6 km in the same time Mandy can ride 4 km.
The girls want to ride 9 km to Fish Creek Park and arrive at the same time. How much of a head start should Mandy have?

problems like this before.

SHOW AND EXPLAIN ALL YOUR WORK

Understand-

How much of a head start should Mandy have if the wanted to arrive at the same time (9km), (Jennifer rides 6km in the same amount of time Mandy rides 4km).

Plan-

Guess and check

Do-

$$4\overline{)6.0} \quad \begin{array}{r} 1.5 \\ -4\times \\ \hline 20 \end{array}$$

Jen 1.5
Mandy 1

1 1.5	1 1
2 1.5	2 1
3 1.5	3 1
4 1.5	4 1
5 1.5	5 1
6+1.5	6 1
90	7 1
	8 1
	9+1
	9

$$\begin{array}{r} 9 \\ -6\times \\ \hline 3 \end{array}$$

Look Back-

Mandy would need a 3km head start.
Yes this is resonable because I've solved

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Partial* ▼

The student has appropriately restated the problem. The plan is to use "Guess and Check" which is not really the process followed.

It would be helpful if the student explained why s/he chose to divide 6 by 4 (as opposed to subtracting or multiplying). The student could have stated more clearly that there is a ratio of 1.5 to 1 as the comparison of Jennifer to Mandy.

The student appropriately charts Jennifer's and Mandy's rate (without actually labelling the headings). The conclusion is that 9 - 6 = 3 but without the chart headings the explanation is not complete.

The reflection on the solution is one of satisfaction but boredom. This could be because it is a repeated exercise rather than one of using a different process to arrive at a solution.

Jennifer and Mandy like to ride bicycles.
Jennifer can ride 6 km in the same time Mandy can ride 4 km.
The girls want to ride 9 km to Fish Creek Park and arrive at the same time. How much of a head start should Mandy have?

SHOW AND EXPLAIN ALL YOUR WORK

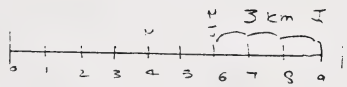
Problem: How much headstart should Mandy have on Jennifer?

procedure *diagram*
ratio = 2:3
LCM
devision into the two number
GCF
check

J : M
6 : 4
3 : 2 Reduce x3
9 : 6

9 − 6 = 3

Answer: 3



= 1 km
M = Mandy
J = Jennifer
Mandy was on the fourth km when Jennifer was on 6.
If Mandy was on the sixth km when Jennifer was on the ninth, ther is a 3 km difference.

answer: Jennifer would have to have a 3 km head start in order for her and mandy to finish at the same time

Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Complete* ▼

The student has restated the problem and has suggested several possible strategies for approaching the problem. S/he has chosen the ratio procedure as the principal strategy, briefly stating REDUCE and x 3 as the process for arriving at a solution.

The check is by means of a diagram but it is incomplete in fully explaining the thought process, although the second sentence could be deemed a check of the answer (If Mandy was on the 6th km ...)

The answer of 3 km head start is accurate; however the statement is that Jennifer needs the 3 km head start rather than Mandy.

This student appears confident in using the two strategies demonstrated. Only the check using the diagram lacks a complete explanation which prevents this from being rated as an elegant response.

**Jennifer and Mandy like to ride bicycles.
Jennifer can ride 6 km in the same time Mandy can ride 4 km.
The girls want to ride 9 km to Fish Creek Park and arrive
at the same time. How much of a head start should Mandy have?**

SHOW AND EXPLAIN ALL YOUR WORK

Problem: How much head start should Mandy Have?

Process: chart
Ratio ✓
L.C.M., G.C.F.
Division into the two numbers

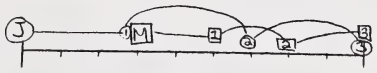
Solve: J : M
6 : 4
↓ ↓
3 : 2
×3 : ×3
↓ ↓
9km : 6 km

$\frac{9}{-6}$
$\frac{}{3}$

Answer: 3 km

Check:

Diagram



or

J	M
0	3
+3	+2
3	5
+3	+2
6	7
+3	+2
9	9

Why: I put Jenifer & Mandy as a ratio (6:4) I put it in lowest form (3:2) I multiplied Jenifer & Mandy's ratio by 3 (3×3:2×3) getting (9:6) This means that if they start at the same spot, by the time Jenifer gets to the end, Mandy will only be at the 6km point. So I subtracted 6 from 9 to find out how much head start Mandy should get. I checked it with a diagram & then an equation. I Jenifer started at 0 and Mandy started at 3, I kept on adding 3 to Jenifer, and 2 to Mandy. The answer came out to 9. So I knew I was right.

▲ *Elegant* ▼

The student has restated the question and has listed several strategic approaches which could lead to the solution. The student appears confident in stating this is a ratio problem and correctly arrives at a correct answer. The check using a diagram is accurately drawn (although it would be considered to be drawn backwards) (???). There is a double-check by using the same ratio process in finding the sum of 9 km for each girl.

The written explanation shows a mature explanation of thinking. This student has demonstrated a creativity in providing several strategic approaches and has concluded aptly - "So I knew I was right."

Circle **one** of the following words you would use to describe this problem:

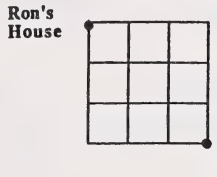
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

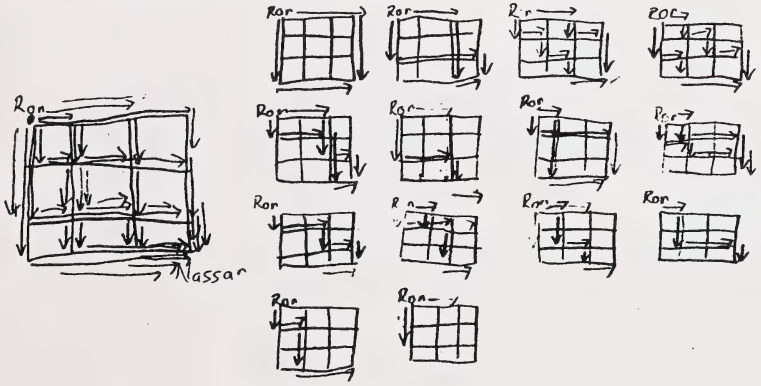
Discouraged *Satisfied* *Frustrated* *Happy*

Grade 8

Ron visits his friend Nassar so often that he likes to vary his route. If Ron never moves diagonally and never walks back toward his own house, how many different routes can he take to go to Nassar's house?

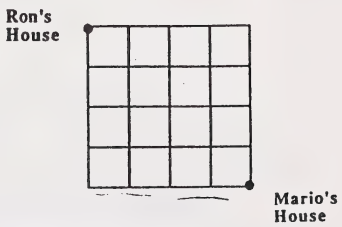


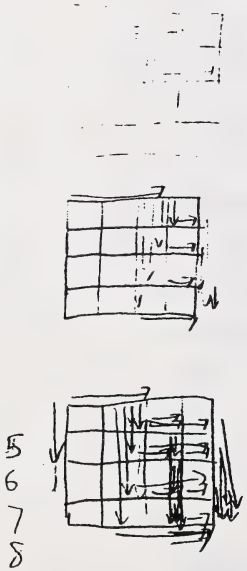
SHOW AND EXPLAIN ALL YOUR WORK



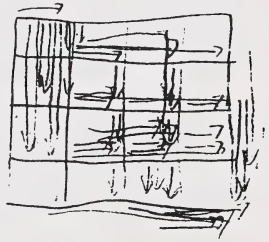
20

Explain how you would determine the different routes Ron could take to go to Mario's house. [Remember Ron never moves diagonally and never walks back toward his own house.]





5
6
7
8
9
10



4
5
6
7
8
9
10
11
12

Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

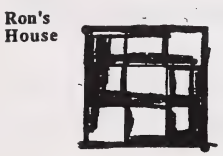
▲ *Partial Complete* ▼

This student has the correct answer but has included no written explanation of why 20 was chosen as a solution rather than 22 in the diagram (some of which appears to be duplicated).

1. *It would be advantageous to interview this student.*
2. *Is the larger drawing (page 1) the initial exploration and the smaller 14 squares the "look back"?*
3. *With some guidance, perhaps the student would have drawn the appropriate reflections in each drawing thus eliminating duplicates.*

Since this appears to be a true problem (where the student has not experienced this problem-solving situation before), the student was unable to recognize the number patterns this problem generated. Had the strategic approach been to develop a similar but simpler problem, S/he might have discovered the myriad of patterns which exist in this question. The perseverance and creativity shown by this student should be commended.

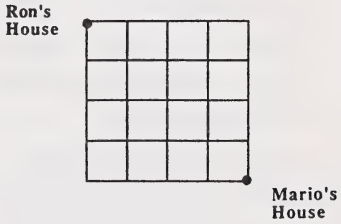
Ron visits his friend Nassar so often that he likes to vary his route. If Ron never moves diagonally and never walks back toward his own house, how many different routes can he take to go to Nassar's house?



SHOW AND EXPLAIN ALL YOUR WORK

There are 20 possible routes

Explain how you would determine the different routes Ron could take to go to Mario's house. [Remember Ron never moves diagonally and never walks back toward his own house.]



I would draw them and count them

▲ *Partial* ▼

It appears this student has developed a strategic approach to arrive at the correct answer of 20, but the process selected (colouring all the routes on one drawing only) limited the clarity of the explanation.

Circle **one** of the following words you would use to describe this problem:

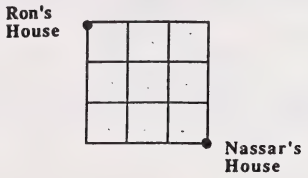
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

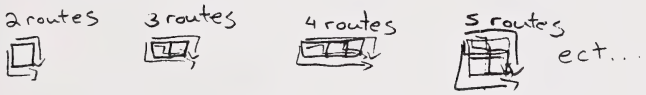
Grade 8

Ron visits his friend Nassar so often that he likes to vary his route. If Ron never moves diagonally and never walks back toward his own house, how many different routes can he take to go to Nassar's house?



SHOW AND EXPLAIN ALL YOUR WORK

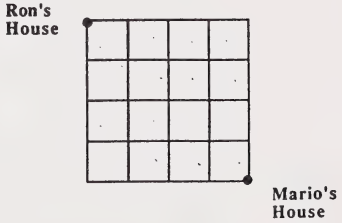
Process: simplify, look for pattern



Answer: 10 routes.

Why: I simplified it to one square. I saw a pattern, one square = 2 routes, two squares = 3 routes, three squares = 4 routes, so I thought if there were nine squares, there must be 10 routes.

Explain how you would determine the different routes Ron could take to go to Mario's house. [Remember Ron never moves diagonally and never walks back toward his own house.]



Process: 16 blocks - 17 routes

Answer: 17 routes

Why: I did it exactly the same way as the first problem.

▲ *Partial* ▼

This student shows an understanding of the problem by choosing a process of simplifying and establishing a pattern. The first three simpler models drawn are indeed correct. It is unfortunate that in the fourth model the student has made an error in counting the number of routes (5 rather than 6).

It is highly probable that without this error, the student would have continued the number pattern so as to arrive at the correct solution.

Because the pattern the student found seemed to work, there was confidence that the answer was correct and therefore the attitudinal response suggested it was fun and there was satisfaction.

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Introduction

We have chosen student work which demonstrates complete responses. Students who went beyond as noted in teacher summaries, provide examples of elegant responses (unusual insight displayed in solution).

All students were motivated since they all satisfied the criteria of the question. Solutions display varying levels of creativity and "straight-forwardness". All were confident in their ability to use mathematics and communicated their ideas well. Strategies used include diagrams, charts, or restatements of the problem.

Students who incorporated the use of algebra over trial and error as a method for solving the problem were ranked as "elegant".

All student work was organized and logical approaches were seen. Teacher comments on each student solution serve to emphasize differences in approach and are made with the assumption that the reader has knowledge of the Math 10 program. Comments are not intended to discredit student work since all examples chosen exhibit high levels of problem solving, but are merely attempts to differentiate between degrees of problem solving shown.

Student work is presented in graduated levels of "elegance" as seen by the writer. Each problem should be read as a set instead of taking each student solution in isolation.

Grade 10

525 can be written as the sum of consecutive numbers.

A. **Find a set of consecutive numbers that will work.**

SHOW AND EXPLAIN ALL YOUR WORK

Understanding the problem

525 must be broken down into a set of consecutive numbers.
There are probably many possible sets.
How many numbers are there in a set?

Plan

525 is an odd number. That means if we decide one set is composed of two numbers, in dividing 525 by 2 we won't get whole numbers.

525 ÷ 2 = 262.5, which means 262.5 + 262.5 = 525

By subtracting .5 from the first number and adding it to the second the two numbers equal to 525 became 262+263. Because this is addition, this modification hasn't affected the operation.

One set of consecutive numbers equal to 525 is 262+263

B. **Can you find another set?**

To find another set I can use the same process as I used before only dividing by 3 instead of 2.

525 ÷ 3 = 175, or 175+175−175 = 525. By subtracting 1 from the first term and adding it to the last term, and leaving the middle term as it is, I get 174+175+176 = 525 Once again, this change to the terms doesn't affect the answer.

Another set of consecutive numbers equal to 525 is 174+175+176.

C. **How many sets of consecutive numbers can you find that add up to 525?**

By following the same model and changing the number 525 is divided by, we can continue to discover more sets. To do this I will now start using a calculator

525 ÷ 4 = 131.25
130 + 131 + 132 + 133 = 526
This one won't work

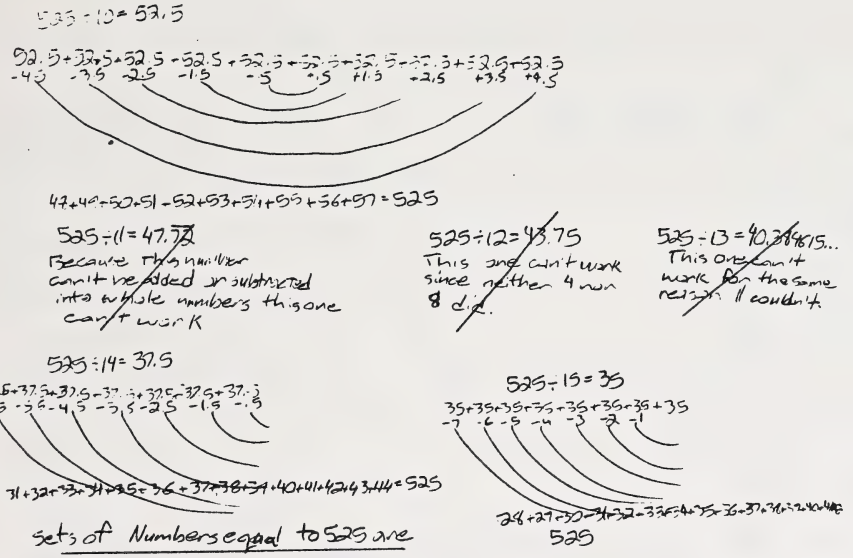
525 ÷ 5 = 105
105 −2, −1, +1, +2
103 + 104 + 105 + 106 + 107

525 ÷ 6 = 87.5
87.5 −2.5, −1.5, −.5, +.5, +1.5, +2.5
85 + 86 + 87 + 88 + 89 + 90 = 525

525 ÷ 7 = 75
75 −3, −2, −1, +1, +2, +3
72 + 73 + 74 + 75 + 76 + 77 + 78 = 525

525 ÷ 8 = 65.625
Because 525 ÷ 4 didn't work this one won't either.

525 ÷ 9 = 58.33
This one won't work because it can't be added or subtracted into whole numbers.



Circle **one** of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

▲*Complete*▼

- *Student is motivated since s/he satisfies all criteria of problem and uses a strategic approach: knows what is needed to solve the problem.*
- *Uses a trial and error approach.*

525 can be written as the sum of consecutive numbers.

A. Find a set of consecutive numbers that will work.

SHOW AND EXPLAIN ALL YOUR WORK

Set 1. If you use negative numbers, the set (-524, -523, -522, ... 523, 524, 525) will add up to 525 with the negatives canceling out the positives.

B. Can you find another set?

Yes. Set 2 525 divided by two is 262.5 Therefore the numbers that average to that are 262 and 263

C. How many sets of consecutive numbers can you find that add up to 525?

Set 3 $525 \div 3 = 175$ so 174, 175, 176

Set 4 (-261, -260, -259, ..., 261, 262, 263) will also equal 525 see set 1

Set 5 (-173, -172, -171 ..., 173, 174, 175, 176) will also equal 525 (see set 1)

Set 6 $525 \div 5 = 105$ so 103, 104, 105, 106 and 107

Set 7 (-102, -101, -100, ..., 103, 104, 105, 106, 107) See set 1

Set 8 $525 \div 7 = 75$, so 72, 73, 74, 75, 76, 77, 78 will work.

Set 9 (-71, -70, -69, ..., 76, 77, 78) see set 1

Set 10 $525 \div 10 = 52.5$ so 48 49 50 51 52 53 54 55 56 57

Set 11 (-47, -46, -45, ... 55, 56, 57) works (see set 7)

Set 12 $525 \div 14 = 37.5$ so 31 ... 44 works

Set 13 (-30, -29, -28, ..., 41, 42, 43, 44) works

Set 14 525 ÷ 5 = 87.5 so : 85, 86, 87, 88, 89, 90 work

Set 15 So does (-84, -83, -82, .. 88, 89, 90)

Set 16 9 to 33.. works. (525 ÷ 21)

Set 17 (-8, -7, -6, ... 30, 31, 32, 33) does too.

Set 18 525 ÷ 15 = 35 so 28; .. 42

Set 19 You know by how that (-27 ... 42) also works.

Set 20 3 ... 32 (525 ÷ 30) works

Set 21 -2 ... 32 works Set 23 -14 ... 35

Set 22 15 to 35

All calculations on calculator or in head

▲*Complete*▼

- *Intriguing first example because the student went beyond initial self-imposed boundaries felt by most students through inclusion of negatives. This shows creativity on part of student.*
- *Demonstrates perseverance in finding 22 solutions to the problem (also makes good use of the hour time allotment). This shows the motivation of the student, also reflected in the attitudinal response at the end of the problem. -Student imposed order on structure to the solution of the problem in the answer in an ordered way - knew and developed a pattern.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* (*Fun*)

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* (*Happy*)

Grade 10

525 can be written as the sum of consecutive numbers.

A. Find a set of consecutive numbers that will work.

SHOW AND EXPLAIN ALL YOUR WORK

$y+(y+1)=525$
$2y=524$
$y=262$
$A=\{262,263\}$

B. Can you find another set?

$x+(x+1)+(x+2)=525$
$3x=522$
$x=174$
$A=\{174,175,176\}$

I found 11 sets of numbers that work. They are all consecutive numbers. Not one part is repeated.

C. How many sets of consecutive numbers can you find that add up to 525? 11

$x+(x+1)+(x+2)+(x+3)=525$
$4x=519$
$x=$ ~~0~~ ← odd won't work with 4.

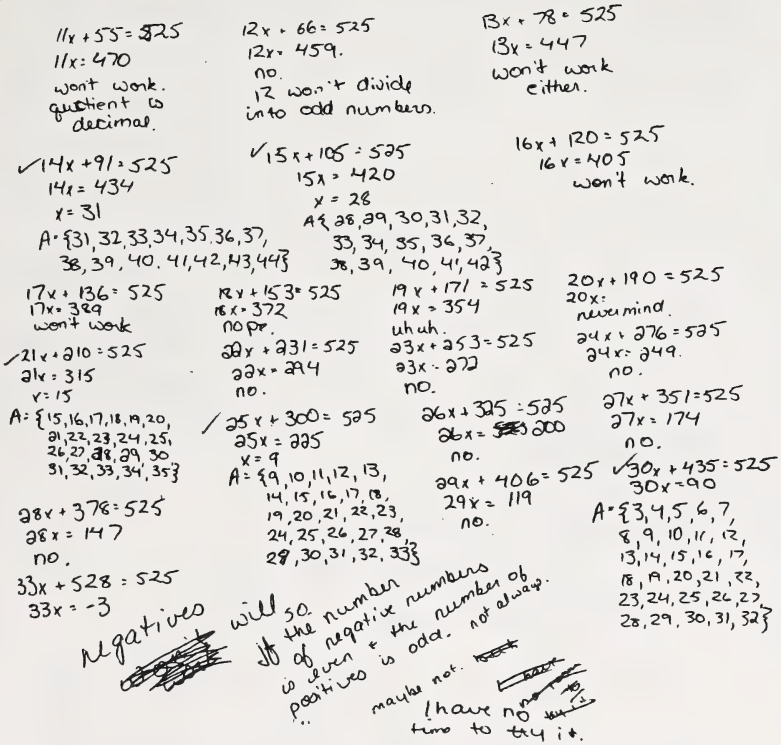
$x+(x+1)+(x+2)+(x+3)+(x+4)=525$
$5x=515$
$x=103$
$A=\{103,104,105,106,107\}$

$8x+28=525$
$8x=497$ won't work ← odd
$9x+36=525$ add 9
$9x=489$
won't work.
result is a decimal.

$x+(x+1)+(x+2)+(x+3)+(x+4)+(x+5)=525$
$6x+15=525$
$6x=510$
$x=85$
$A=\{85,86,87,88,89,90\}$

$x+(x+1)+(x+2)+(x+3)+(x+4)+(x+5)+(x+6)=525$
$7x+21=525$
$7x=504$
$x=72$
$A=\{72,73,74,75,76,77,78\}$

$10x+45=525$
$10x=480$
$x=48$
$A=\{48,49,50,51,52,53,54,55,56,57\}$



Circle one of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

waste of time

Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲*Complete*▼

- *Student demonstrates an understanding of the problem.*
- *Uses algebra to begin, noting the simplest cases (in A and B).*
- *Student tries a large number of cases.*
- *A very systematic and persistent solution.*
- *The student develops more efficient ways, after the 7th trial.*

525 can be written as the sum of consecutive numbers.

A. Find a set of consecutive numbers that will work.

SHOW AND EXPLAIN ALL YOUR WORK

$525/2 = 262.5 \rightarrow 262+263 = 525$

$525/15 = 35 \rightarrow$ 27+28+29+30+31+32+34+35+36+37+38+39+40+41+42 = 525

$\frac{525}{x} = y$, y is the average of x consecutive numbers adding up to 525

eg: $\frac{525}{5} = 105$. y must be a number ending in 0.5 or .0, for the numbers to be consecutive + add up to 525

If $\frac{525}{x} = y$, and x is odd, then the consecutive #s are: $q = \frac{x-1}{2}$

$y-q, \ldots, y-(q-(q-1)), y, y+(q-(q-1)) \ldots y+q$

eg: $\frac{525}{7} = 75$, $\rightarrow$ 72, 73, 74, 75, 76, 77, 78

B. Can you find another set?

$525/3 = 175 \rightarrow 174+175+176 = 525$

$525/5 = 105 \rightarrow 103+104+105+106+107 = 525$

$525/6 = 87.5 \rightarrow 85+86+87+88+89+90 = 525$

$525/7 = 75 \rightarrow 72+73+74+75+76+77+78 = 525$

C. How many sets of consecutive numbers can you find that add up to 525? (42) (using negative numbers as well as positive)

525 ÷ 2, 3, 5, 6, 7, 10, 14, 15, 21, 25, 30, 35, 42, 50, 70, 75, 105, 150, 175, 210, 350

$525/x = y$		will/will not give consecutive numbers adding to 525
2	262.5	will
3	175	will
4	131.25	will not
5	105	will
6	87.5	will
7	75	will
8	65.625	will not
9	$58.\overline{3}$	NO
10	52.5	YES
11	$47.\overline{7272}$	NO
12	43.75	NO
13	40.3846	NO
14	37.5	YES
15	35	YES
21	25	YES
25	21	YES
30	17.5	YES

▲*Complete* ▼

- *Student immediately demonstrates an understanding of the problem with two examples that worked.*
- *Student differentiates between odd and even combinations.*
- *Student generalizes "rules" for the different cases.*
- *The student is persistent in probing more deeply into the problem.*
- *The student continues by organizing a series of cases, and then makes a table.*
- *It appears that the solution in this paper is only part of the student's attempt to solve the problem.*

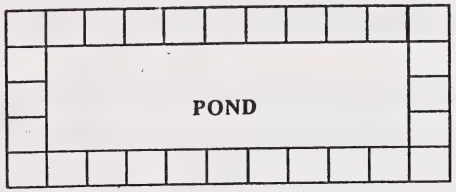
Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

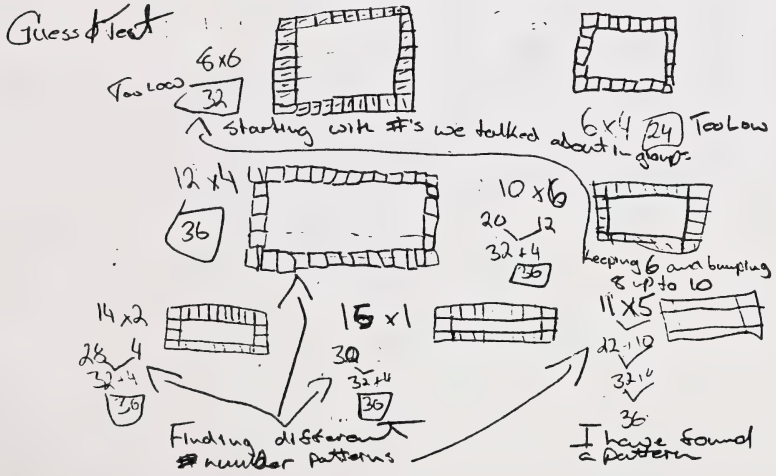
Circle **one** of the following words that describes how you feel after trying to solve this problem:

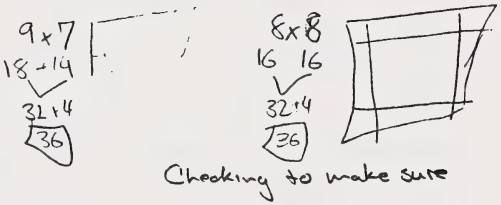
Discouraged *Satisfied* *Frustrated* *Happy*

Paving stones that measure 1 x 1 are used to make a border around a rectangular pond.
In the example below the pond is 3 x 9 and 28 stones are used.



What are the different dimensions of ponds that would require 36 paving stones to complete the border?





Conclusion:

After working with different numbers trying to get 36 paved stones ~~or~~ built around a pond, the different Combinations I ~~a~~ have came up with are

8~~x~~8 = 16+16 = 32+4 (outside corners) = 36
9 by 7 = 18+14 = 32+4 (" ") = 36
10 by 6 = 20+12 = 32+4 (" ") = 36
11 by 5 = 22+10 = 32+4 (" ") = 36
12 by 4 = 24+8 = 32+4 (" ") = 36
13 by 3 = 26+6 = 32+4 (" ") = 36
14 by 2 = 28+4 = 32+4 (" ") = 36
15 by 1 = 30+2 = 32+4 (" ") = 36

The lowest possible combination is 8 by 8
The highest possible Combination is 15 by 1, Because 16 by 0 cannot work because then their would be no pond eg 16 by 0.

that would be the answer It does not have 36 paved stones

▲ *Complete* ▼

- *Student selects and names the strategy used.*
- *Student uses a group discussion as a springboard to begin the problem. -Guesses are examined for appropriateness and conclusions are drawn ("too low"). -Student generalizes and states a pattern using the numbers.*
- *Self-monitoring is evident in the process of solving the problem. -Student completes the problem by making an organized list and checking. -Further, student presents an argument to support the results.*

Circle **one** of the following words you would use to describe this problem:

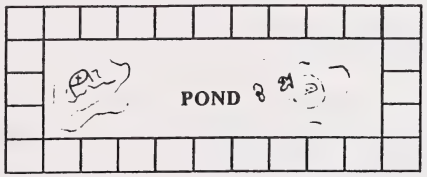
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Grade 10

Paving stones that measure 1 x 1 are used to make a border around a rectangular pond.
In the example below the pond is 3 x 9 and 28 stones are used.



What are the different dimensions of ponds that would require 36 paving stones to complete the border?

SHOW AND EXPLAIN ALL YOUR WORK

Understanding. To My knowledge, this pond shown is 9x3, and is 28 all the way around. Somehow I must figure out a formula to find the ammount of stones to the dimensions.

9x3 are the dimensions if you don't include the corners. when you multiply the sides of a rectangle you get the tottal # of units. But 9x3 = 27. thats one short. Maybe if you add one you can get the answer.

The problem is I'm not sure what I think. but I found the correct way to find the dimensions using trial and error.

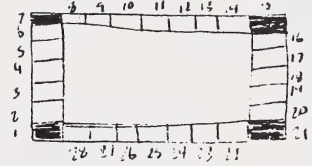
Developing $(1x^1+y^1)+1=T$

X and Y are sides and the T is the total. This might work If I dont count the corners. Block them out. Besides, they wouldnt Really be touching the matter.

If this equasion works a possibele sollution for T = 36 is 5x7. Because 5x7 is 35 +1 equals 36.

Now to try it.

carrying out plan.



Obviously there are not enough blocks. So this plan did not work.

★ These all equal demensions of Pond. Big mistake.

Developing

$$1x+4^1x2+4=T$$

This time I found that If you add 9+8 you get 17. x2 equals 34 add the corners gives you 38.

All I simply have to do is work backwards $\frac{36-4}{2}=16$.

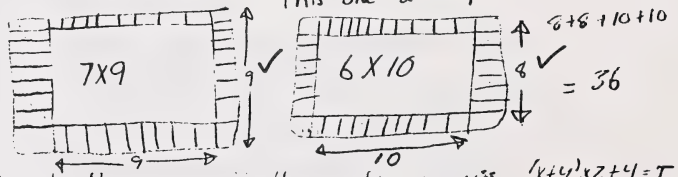
Now just get 2 numbers that go into 16.

I picked 7+9.

$$(7+9)^1x2+4=36$$

Now to try it.

Carrying out



Conclusion. To get the answer to this question you use $(x+y)^1x2+4=T$.

In this one were T=36. I worked backward then I got x+4=16.

Till the possibele comparations add up to 16.

★ The different dimensions of the pond with 36 paving stone are 2X14, 3X13, 4X12, 5X11, 6X10, 7X9, 8X8.

★→ The dimensions of the stones are.
16x2, 3x15, 4x14, 5x13, 6x12, 7x11, 8x10, and 9x9.

▲*Complete* ▼

- *Adopts the strategy of working backwards.*
- *Missing 15 X 1 solution.*
- *Student clarifies the information given in the problem.*
- *Student communicates the thinking clearly.*
- *Student is persistent in exploring the solution to the problem.*
- *Student demonstrates flexibility by beginning with an unsuccessful plan then selecting a new plan. - Student generalizes using an algebraic expression.*
- *Once the student arrives at a workable plan, solutions to the problem are quickly developed.*

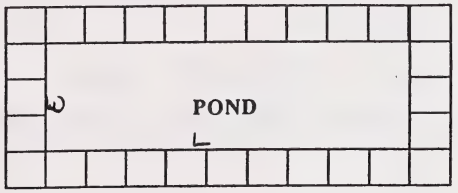
Circle **one** of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* (*Frustrated*) *Happy*

Paving stones that measure 1 x 1 are used to make a border around a rectangular pond.
In the example below the pond is 3 x 9 and 28 stones are used.



What are the different dimensions of ponds that would require 36 paving stones to complete the border?

SHOW AND EXPLAIN ALL YOUR WORK

1) Understanding.
1) Using 36 stones, what are the dimensions of the pond not the border!

2) The dimensions are of the inside (-the pond) not the outside (-the border)

2) Plan
Let Dimension = W+1 x L-1

3.) Carrying out the plan
36 (3 on each side)
-6
30 ÷ 2 = 15
1st dimension = 1 x 15
2nd dimension = 1+1 x 15-1
2 x 14
3rd = 3 x 13
4th = 4 x 12
5th = 5 x 11
6th = 6 x 10
7th = 7 x 9
8th = 8 x 8 (not a square because of the corner stones!)
9th = 9 x 7
10th = 10 x 6
11th = 11 x 5
12th = 12 x 4
13th = 13 x 3
14th = 14 x 2
15th = 15 x 1

As the numbers reverse the rectangle goes from this position: ▭ to this position: ▯.

4) Looking Back:

when using $D = w+1 \times L-1$ the position of the original rectangle changes so in fact there are only 8 different dimensions. Once the numbers begin to reverse, the dimensions repeat themselves and $w \times L$. reverses to $l \times w$.

Circle one of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

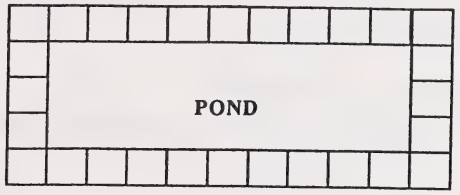
Circle one of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

▲ *Complete* ▼

- *Recognizes importance of pond dimensions not the border.*
- *Clearly states problem approach.*
- *Demonstrates confidence in solution.*
- *Strong visualization skills demonstrated without supporting diagrams (see conclusions). -A novel solution.*
- *Clarified key ideas in the problem.*
- *Created a strategy to generate the dimensions of the pond using the original.*
- *Used algebraic symbols (w and l) to generalize the procedure.*
- *Student recognizes a pattern in the numbers and describes it.*

Paving stones that measure 1 x 1 are used to make a border around a rectangular pond.
In the example below the pond is 3 x 9 and 28 stones are used.



What are the different dimensions of ponds that would require 36 paving stones to complete the border?

SHOW AND EXPLAIN ALL YOUR WORK

Time for a table.

A	B	Blocks
1	3	12
2	6	20
3	9	28
4	12	36

How I got this is simple. The dimensions are on the inside and don't include the corner stones. So you have to multiply A and B by 2 and then add 4 for the corner stones.

Time to set a pattern.
2A+2B+4=stones

Let's list dimensions that work.
① 4×12 (4+2)+(12×2)+4=36
subtract 4 to get the required sum of 28.
36-4=32
32 - even number ① - odd number ②
Divide the even numbers by two to get the dimensions.

① 32 - 16 = 16 ; 16 - 8 = 8
② 32 - 16 = 16 ; 16 - 8 = 8
③ 32 - 30 = 2
④ 32 - 26 = 6
⑤ 32 - 24 = 8
⑥ 32 - 22 = 10
⑦ 32 - 20 = 12
⑧ 32 - 14 = 18
⑨ 32 - 12 = 20
⑩ 32 - 10 = 22
⑪ ⑫

After 16 & 16, they start to repeat.

Dimensions
① 1×15 ② 14×2 ③ 13×3 ④ 12×4 ⑤ 11×5 ⑥ 10×6 ⑦ 9×7
⑧ 8×8

▲ *Complete* ▼

- *Selects a strategy to organize the information.*
- *Uses variables to write a mathematical statement to describe the relationships in the problem (then explains the relationship in words). -Uses mathematical vocabulary and terminology comfortably ("time for a table", "even numbers")*
- *Very confident ("this is simple").*
- *Persistent in showing many cases, then recognizing a repeating pattern.*
- *Used the pattern to generate and record all of the cases.*
- *It would appear that the student, by underlining each case, was systematically checking the work.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

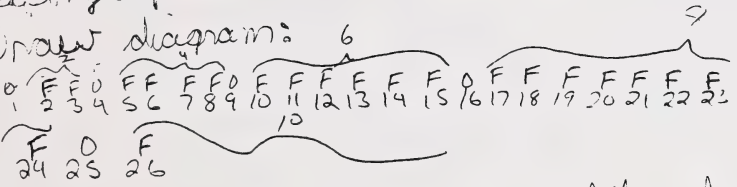
Grade 10

A spaceship control panel has 1000 lights with individual OFF/ON switches. The switches are numbered consecutively from 1 to 1000. All switches are in the OFF position. One thousand cadets are waiting in another room. The first cadet enters the control room and switches on every light. The second cadet comes in and changes the position of every second switch. The third cadet comes in and changes the position of every third switch. Which lights would be ON after the thousandth cadet has gone through?

SHOW AND EXPLAIN ALL YOUR WORK

1000 lights
1000 people
1 person turn on all light
2 person turn on/off every 2nd light
3 person turn on/off every 3rd light
Which light will be on after the 1000 person.

Drawing a plan:
Draw diagram:



I see every on switch is follow by an even number of off switch.

Number give + one even number + 1 = next switch on.

25 + 10 + 1 = 36
36 + 12 + 1 = 49
49 + 14 + 1 = 64
64 + 16 + 1 = 81
81 + 18 + 1 = 100

I see the given number on add a following odd number equal to the next on switch

100 + 21 = 121
121 + 23 = 144
144 + 25 = 169
169 + 27 = 196
196 + 29 = 225
225 + 31 = 256
256 + 33 = 289
289 + 35 = 324 + 37 = 361 + 39 = 400 + 41 = 441 + 43 = 484 + 45 = 529
529 + 47 = 576 + 49 = 625 + 51 = 676 + 53 = 729 + 55 = 784
784 + 57 = 841 + 59 = 900 + 61 = 961

Conclusion: all 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529 (576) 625, 676, 729, 784, 841, 900, 961 are turn on

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

▲ *Complete* ▼

- *Student explained an understanding of the conditions of the problem.*
- *Mention of an even number/odd number rule. Discovered a pattern.*
- *Student uses novel pattern to generate perfect squares.*
- *Selected a strategy (diagram) and indicated a pattern in the "off" switches.*
- *The student was persistent in following the process through to a complete solution.*
- *A statement is included to answer the question asked.*
- *The student clearly communicated the thinking.*

A spaceship control panel has 1000 lights with individual OFF/ON switches. The switches are numbered consecutively from 1 to 1000. All switches are in the OFF position. One thousand cadets are waiting in another room. The first cadet enters the control room and switches on every light. The second cadet comes in and changes the position of every second switch. The third cadet comes in and changes the position of every third switch. Which lights would be ON after the thousandth cadet has gone through?

SHOW AND EXPLAIN ALL YOUR WORK

1. 10 cadets

OFF	N	N	N	N

1. Y Y Y Y Y Y Y Y Y Y
2. Y N Y N Y N Y N Y N
3. Y N N N Y Y Y N N N
4. Y N N Y Y Y Y Y N N
5. Y N N Y N Y Y Y N Y
6. Y N N Y N N Y Y N Y
7. Y N N Y N N N N N Y
8. Y N N Y N N N N Y Y
9. Y N N Y N N N N Y N
10. Y N N Y N N N N Y N

1 2 3 4 5 6 7 8 9 10 11 12 13

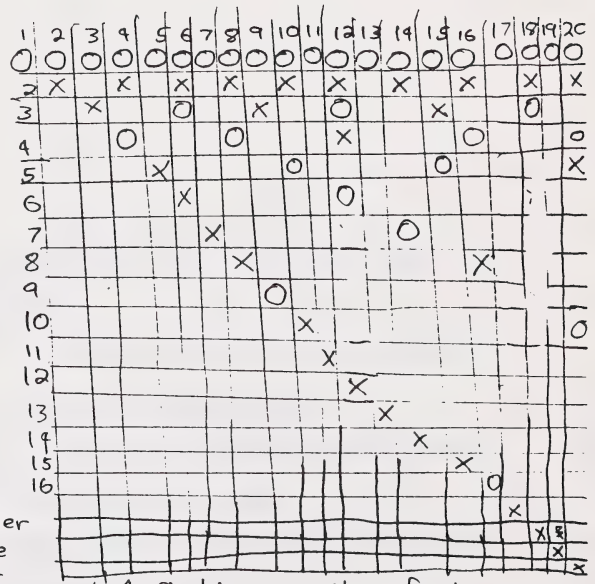
After each position is changed by it's numbers corresponding cadet, it shan't be changed another time ever again.

All prime numbers would be off. They have no factors

36
2 18
2 9
3 3
3

Factors of
4. 2·2 16. 4·4 or 2^4
8. 2·2·2
9. 3·3
10. 2·5

All ~~have~~ an even number of prime ~~factors which are equal to each~~ other
All have two factors equal to each other



1, 4, 9, 16 are all perfect squares, so all perfect squares will be on.

$\sqrt{1000}$ = under 32 because $32^2 = 1024 = 2^{10}$

IKB

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961 will all be on.

▲ *Complete* ▼

- *First paragraph clearly shows understanding of problem.*
- *The student began by solving a simpler problem, then saw a pattern.*
- *Student used the language of mathematics to clearly communicate observations (prime numbers; perfect squares). -A grid was used as a model to further illustrate the pattern of perfect squares.*
- *As a check, the student indicated the limitations (under 32 or 1,000) to prove their solution was correct.*

Circle **one** of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

A spaceship control panel has 1000 lights with individual OFF/ON switches. The switches are numbered consecutively from 1 to 1000. All switches are in the OFF position. One thousand cadets are waiting in another room. The first cadet enters the control room and switches on every light. The second cadet comes in and changes the position of every second switch. The third cadet comes in and changes the position of every third switch. Which lights would be ON after the thousandth cadet has gone through?

SHOW AND EXPLAIN ALL YOUR WORK

Person 1

Off

On 1 | | | | | | | | | | | | | | | | The first one will always be on.

on X off

Light 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Person 1

2 X X X X X X X X

3 X X X X X X X X

4 X X X X X X X X

5 X X X X X X X X

6 X X X X X X X X

7 X X X X X X X X

8 X X X X X X X X

9 X X X X X X X X

10 X X X X X X X X

11 X X X X X X X X

12 X X X X X X X X

13 X X X X X X X X

14 X X X X X X X X

15 X X X X X X X X

16 X X X X X X X X

I noticed the 2 off 1 on 4 off 1 on 6 off 1 on after and found that the numbers were all perfect squares.

Notes: All prime numbers turn OFF. because they have no factors so on their turn they will be turned, left on ? no others. All numbers of 3 factors and one or any odd # of factors will be turned off

All perfect □s will be on.

This is because they have one set of factors which is a number times itself. This number only counts once, so there will be an odd number of factors e.g. on-off-on-off-on

This is the only way to get an odd # of factors.

on

25 x 4

100 x 1

50 x 2

5 x 20

10 x 10 = 10 so 9 factors LIST

off

4 x 8

2 x 16

1 x 32

6 factors so off.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196,

225, 256, 289, 324, 361, 400, 441, 484, 529,

576, 625, 676, 729, 784, 841, 900, 961,

For score and seven years ago I did a similar problem about lockers with 1000 lockers open or closed.

▲ *Complete* ▼

- *Uses math terminology.*
- *Use of generalization (seen in student's notes).*
- *The student creates a very organized chart/diagram to model the information in the problem.*
- *The student discovers a pattern and then elaborates by connecting ideas about even numbers of factors.*
- *The student lists all possibilities and then "looks back" at a similar problem (but in the context of lockers). -Student connected past experience to this situation.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

A. Take any four consecutive numbers; multiply them together and add 1.
Take the square root of the number.
Is your final answer a whole number?

SHOW AND EXPLAIN ALL YOUR WORK

Understanding Problem

1. multiply 4 consecutive numbers, add one, then find square root
2. multiply middle two numbers of four consecutive numbers and compare.
3. Write down possible rule

Develope and Carry out Plan

1. choose numbers, and follow steps. compare!

1. $3 \times 4 \times 5 \times 6 = 360$ → 4 consecutive numbers
$360 + 1 = 361$ → add one
$\sqrt{361} = 19.$ → take square root of number
Yes this is a whole number.

B. Take the middle pair of numbers and multiply them together.
Compare with your first answer.

2. 4×5 → multiply middle two numbers.

ANSWER - 20

The first answer is one less than the second answer $20 - 1 = 19$

or

The second answer is one more than the first answer $19 + 1 = 20$

TRY ANOTHER ONE!

$10 \times 11 \times 12 \times 13$ → multiply 4 consecutive numbers.
$17160 + 1$ → add one
$\sqrt{17161} = 131$ → square root of number.
11×12 → multiply middle two numbers.
132.

compare { $131 + 1 = 132$; $132 - 1 = 131$ }

C. Repeat parts A and B three more times and write down a possible rule.

1. $9 \times 10 \times 11 \times 12 = 11880$
$11880 + 1 = 11881$
$\sqrt{11881} = 109$
$10 \times 11 = 110$
compare { $109 + 1 = 110$; $110 - 1 = 109$ }

2. $25 \times 26 \times 27 \times 28 = 491400$
$491400 + 1 = 491401$
$\sqrt{491401} = 701$
$26 \times 27 = 702$
compare { $701 + 1 = 702$; $702 - 1 = 701$ }

3. $19 \times 20 \times 21 \times 22 = 175560$
$175560 + 1 = 175561$
$\sqrt{175561} = 419$
$20 \times 21 = 420$
compare { $419 + 1 = 420$; $420 - 1 = 419$ }

Conclusions

RULE - When you find the square root of four consecutive numbers plus one, the answer will be one less than the product of the 2 middle numbers of the 4 consecutive numbers.

▲ *Complete* ▼

- *Written explanation to describe the mathematics used for the rule (as opposed to the use of algebra for the rule). -Student read the whole problem then describes what is to be done in understanding the problem.*
- *The plan is carried out in a very organized manner.*
- *The possible rule is stated as a conclusion to the problem.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

A. Take any four consecutive numbers; multiply them together and add 1.
Take the square root of the number.
Is your final answer a whole number?

SHOW AND EXPLAIN ALL YOUR WORK

① $1 \cdot 2 \cdot 3 \cdot 4 + 1 = 25 \quad \sqrt{25} = 5$

② $10 \cdot 11 \cdot 12 \cdot 13 + 1 = 17161 \quad \sqrt{17161} = 131$

③ $25 \cdot 26 \cdot 27 \cdot 28 + 1 = 491\,401 \quad \sqrt{491\,401} = 701$

④ $19 \cdot 20 \cdot 21 \cdot 22 + 1 = 175561 \quad \sqrt{175\,561} = 419$

Yes, the final answer is a whole number.

B. Take the middle pair of numbers and multiply them together.
Compare with your first answer.

① $2 \cdot 3 = 6$
② $11 \cdot 12 = 132$
③ $26 \cdot 27 = 702$
④ $20 \cdot 21 = 420$

These answers are all one more than in the first section.

C. Repeat parts A and B three more times and write down a possible rule.

→ $100 \cdot 101 \cdot 102 \cdot 103 + 1 = 106\,110\,601 \quad \sqrt{106\,110\,601} = 10\,301$
$101 \cdot 102 = 10\,302$

→ $30 \cdot 31 \cdot 32 \cdot 33 + 1 = 982081 \quad \sqrt{982\,081} = 991$
$31 \cdot 32 = 992$

→ $27 \cdot 28 \cdot 29 \cdot 30 + 1 = 657\,721 \quad \sqrt{657\,721} = 811$
$28 \cdot 29 = 812$

You take four consecutive number and multiply the two middle ones together, then subtract one.
a, b, c, d $b \cdot c - 1$
It will be the same answer as multiplying all the numbers, adding one and square rooting it.

$\sqrt{a \cdot b \cdot c \cdot d + 1} = b \cdot c - 1$

▲*Complete* ▼

- *Student tried four examples in parts A and B then wrote a conclusion in each case.*
- *Student used more trials then described in writing the rule.*
- *In conclusion, the rule was stated in algebraic form.*

Circle **one** of the following words you would use to describe this problem:

Boring (*Challenging*) *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged (*Satisfied*) *Frustrated* *Happy*

Three CTS buses leave a south Calgary bus stop for downtown at the same time following different routes. The first bus takes 20 minutes to make the round trip, the second takes16 minutes and the final bus takes 12 minutes. How many round trips would the first bus make until all three buses are at the same stop again? Explain your answer.

SHOW AND EXPLAIN ALL YOUR WORK

The next multiple of 20 that is divisible by, both 16 and 12 will be the next time they are at the stop at the same time. Let's try a chart of multiples

The First number that is found in all 3 columns is the next time they are at a stop together.

	20	16	12
1	20	16	12
2	40	32	24
3	60	48	36
4	80	64	48
5	100	80	60
6	120	96	72
7	140	112	84
8	160	128	96
9	180	144	108
10	200	160	120
11	220	176	132
12	240	192	144
13	260	208	156
14	280	224	168
15	300	240	180
16	320	256	192
17	340	272	204
18	360	288	216
19	380	304	228
20	400	320	240

In 240 minutes, they will be at the same stop again. Now, the number in the 12 column is the answer.

The First bus will make 12 round trips before the three buses are at the same stop.

▲ *Complete* ▼

- *Student uses a table to organize the trials.*
- *The reason for using a chart is stated.*
- *Good explanation of strategy.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

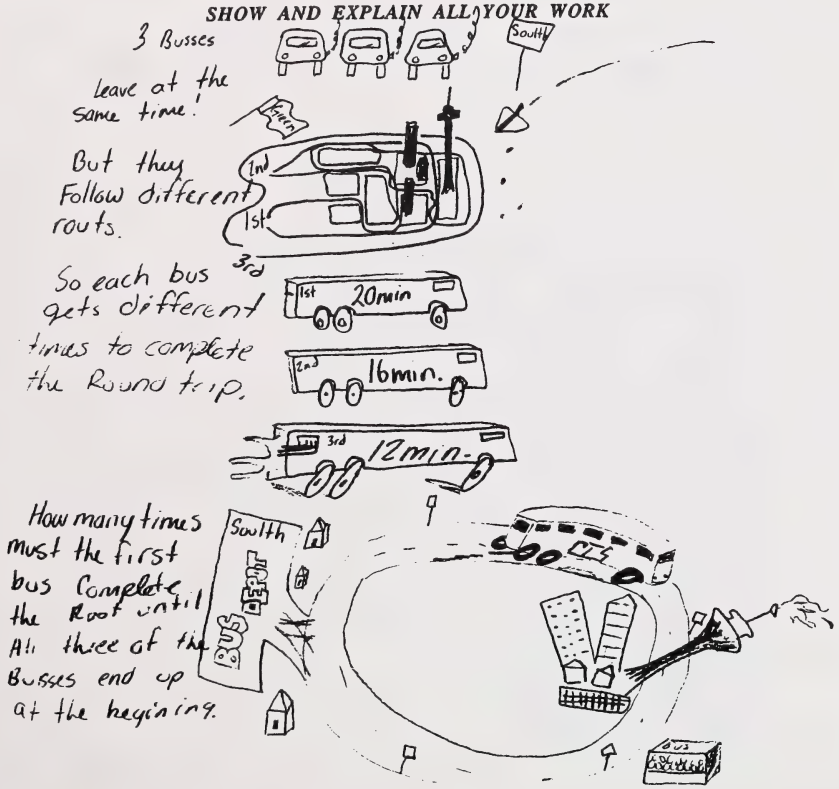
Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Grade 10

Three CTS buses leave a south Calgary bus stop for downtown at the same time following different routes. The first bus takes 20 minutes to make the round trip, the second takes16 minutes and the final bus takes 12 minutes. How many round trips would the first bus make until all three buses are at the same stop again? Explain your answer.

SHOW AND EXPLAIN ALL YOUR WORK


3 Busses
Leave at the same time!
But they Follow different routs.
So each bus gets different times to complete the Round trip.
How many times must the first bus Complete the Root until All three of the Busses end up at the begining.
South
1st
2nd
3rd
20min
16min.
12min.
BUS STOP

This question appears to be very simple. the First plan that I would attempt is to: Take the three numbers 20, 16, and 12. And find the Lowest common multiple. By doing this I would find the first time that they would meet at the same stop But to get the answer I would divide the LCM By 20 to get the number of TIMES the first buss took its trip.

20	16	12
40	32	24
60	48	36
80	64	48
100	80	60
120	96	72
140	112	84
160	128	96
180	144	108
200	160	120
220	176	132
240	192	144
260	208	156
	224	168
	240	180
	256	192
		204
		216
		228
		240

The lowest common multiple From these three numbers is 240. Now I divide 240 by 20 to get the answer. Which is $\frac{20}{240} = 12$.

★ If three CTS Busses leave a point at the same time. And they each are going in different Routs. With the busses taking 20, 16, and 12 min. to complete the Round trips. The bus that takes 20 min. Will go 12 Routes before It starts all over with the other two busses at the beginning.

▲*Complete*▼

- *It seems that the student enjoyed sorting out the information in the problem.*
- *The problem is very simple to the student yet he/she explains the thinking and procedure in great detail. -Clear and concise.*
- *Mention of LCM.*
- *Good explanation and great art work.*
- *Chart reflects the student's thought process in looking for the LCM.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Three CTS buses leave a south Calgary bus stop for downtown at the same time following different routes. The first bus takes 20 minutes to make the round trip, the second takes16 minutes and the final bus takes 12 minutes. How many round trips would the first bus make until all three buses are at the same stop again?
Explain your answer.

SHOW AND EXPLAIN ALL YOUR WORK

First of all, you must find the lowest common multiple of the 3 #s.

LCM of 20 = $5 \times 2 \times 2$
16 = $2 \times 2 \times 2 \times 2$
12 = $3 \times 2 \times 2$

$4 \times 2 \times 5 \times 2 \times 3 = 240$

Then the 1st bus takes 20 minutes. So: $20 \times ? = 240$
The answer is 12.

Answer: 12 round trips

This is because the LCM determine how many minutes until all 3 buses are at the same stop. For the 1st bus, because it takes 20 minutes for one round trip, then that means that to reach 240 minutes, it would have to take 12 round trips.

▲ *Complete* ▼

- *Use of prime factors to find the LCM.*
- *Immediate solution.*
- *Problem is crystal clear to the student.*
- *Student very quickly realizes the relationship of LCM to this problem and at the end does a nice job of explaining its significance.*

Circle **one** of the following words you would use to describe this problem:

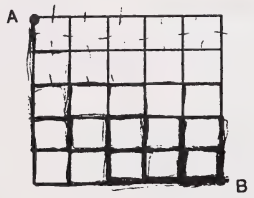
Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

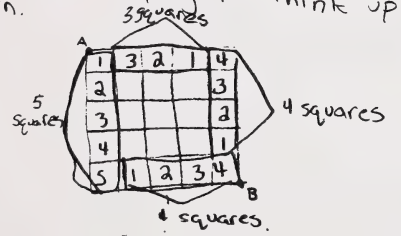
Show how many different paths there are from the Point A to the Point B on a 5 x 5 square if only moves to the right or downward along sides of squares are permitted?

SHOW AND EXPLAIN ALL YOUR WORK



I'm not really sure so my first thing to do would be to just count.

Since there are too many squares and possible routes I'm trying to think up a possible pattern.

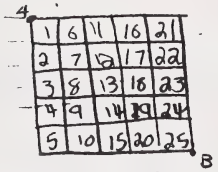


$5 \times 4 \times 4 \times 3 =$
$20 \times 4 \times 3 =$
$80 \times 3 = 240$

Let me rephrase it → So that means that by that way I am guessing that there is a possible of 240 diffrent paths.

So that means there could be a possible of 240 diffrent paths

I'm going to try to think up a diffrent possible pattern.



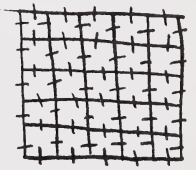
1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

Since there are 25 squares ~~I~~ and this is a big square I'll find the square root of 25 which is 5. so I think that you take 25 and multiply it by 5 and come up with an answer of 125.

So that means that there could be a possible of 125 diffrent paths.

- Now I got two diffrent answers.
- I with a possible 240 diffrent paths.
- The other with a possible 125 diffrent paths.

I think I found another way.



I counted all the lines of the square. There is 60.

There could be a possible of 60 diffrent paths.

I have came up with 3 choices. I choose 125 diffrent paths because it goes the best and it seems more logical

▲*Complete* ▼

- *Trying different strategies, creative.*
- *Persistent.*
- *Attempt at looking back, "Is this answer reasonable"?*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

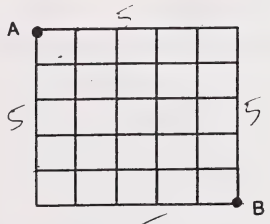
Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Grade 10

Show how many different paths there are from the Point A to the Point B on a 5 x 5 square if only moves to the right or downward along sides of squares are permitted?

SHOW AND EXPLAIN ALL YOUR WORK



10 intersections
10 choices
except for outside choice which has 4

When one travels the path, one has ten choices to get to the end, except if a path is chosen which goes around the perimiter, where there would be 9 choices. Therefore,

2→2→2 O
2→2→2 O
2→2→2 O
O O O O

9 intersections with 2 choices
2 ways to get to 4
1 way to get to 5

2
2

76

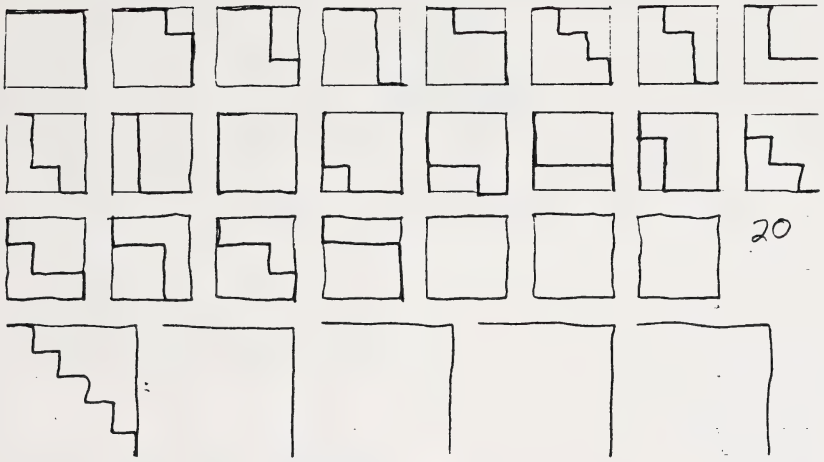
1
2 1 2 choice except for outside (1)
4
10 5 2 4 choices except for outside (3)
14
10 20 3 6 choices except for outside (8)
24
(see graph paper)

$n^n - n^2 + 2$

$6 \times 5 - 8$
$6! - 8$
~~2~~ so -8

10— 42?
34 — —
76?

Number of choices is $2n$ where n is the ~~number of~~ dimension of ~~the~~ each side.



▲*Complete*▼

- *Student appears to understand the problem.*
- *Starts with a simpler problem. Does 1 x 1, 2 x 2, and 3 x 3.*
- *In the 3 x 3 the possible routes were found in a very systematic way. 4 x 4 and 5 x 5 became too complicated but the strategy up to this point was positive. -Example of preliminary response not a complete solution, but one typically used by many students.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Grade 10

Show how many different paths there are from the Point A to the Point B on a 5 x 5 square if only moves to the right or downward along sides of squares are permitted?

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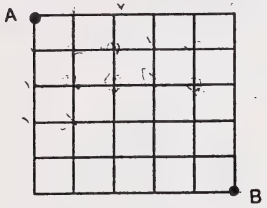
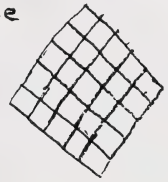
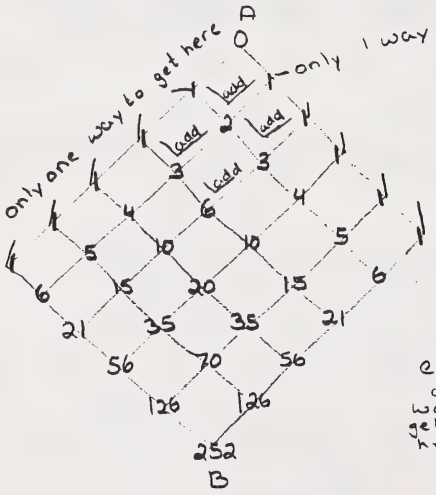


UNDERSTANDING THE PROBLEM

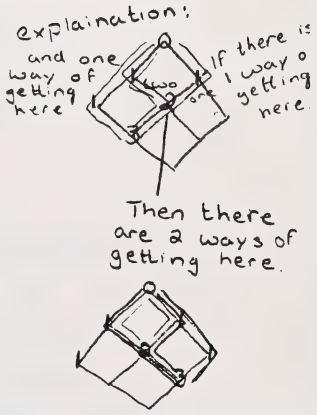
Can only move ↓ and →
Have a 5 x 5 grid
How many different paths

PLAN & DO

If you can only move in one direction turn the grid to look like  Now you can easily trace the paths.



There are 252 ways of getting from Point A - Point B.



▲ *Complete* ▼

- *Student sets up a system for finding the routes to any point then continues it to the end of the 5 x 5 grid.*
- *Takes the time to explain why the pattern works using a simple case.*
- *It would appear that the student has encountered this problem or a similar problem but certainly uses this previous knowledge in solving the problem.*

Circle **one** of the following words you would use to describe this problem:

Boring *Challenging* *Confusing* *Fun*

Circle **one** of the following words that describes how you feel after trying to solve this problem:

Discouraged *Satisfied* *Frustrated* *Happy*

Appendix 1: EQI-Math Project Outline

	Phase 1: *1989 - 1990* DEVELOPMENT	Phase 2: *1990 - 1991* FIELD TESTING	Phase 3: *1991 - 1992* PILOT TESTING
Personnel:	▲ School System Mathematics Specialists (5) ▲ teacher researchers (14)	▲ School System Mathematics Specialists (5) ▲ teacher researchers (14)	▲ School System Mathematics Specialists (5) ▲ teacher researchers (14) ▲ pilot teachers (8)
Tasks:	▲ problem-solving dispositions identified through anecdotal records of problem solvers ▲ problem bank developed ▲ problem format and student attitude questions chosen ▲ holistic scoring studied	▲ 10 problem-solving dispositions tracked ▲ open-ended anecdotal records kept ▲ eight problems administered: choice of time for response and group work ▲ 2 scorings for each problem ▲ student attitude collected (different for all grades) ▲ classroom environment described by teachers	▲ 5 problem-solving dispositions tracked ▲ structured anecdotal records kept ▲ six problems administered: choice of time for response and group work ▲ 2 scorings for each problem ▲ student attitude collected (same for all grades) ▲ classroom environment described by specialists
Results:	▲ 10 dispositions same for all grades: perseverance, risk-taking, confidence, motivated, achieving, understanding, communicative, creative, strategic/ organized, reflective ▲ holistic scoring: 'A' inadequate 'B' adequate 'C' competent	▲ 10 dispositions too unwieldy: analysis helped develop 5: #1 Motivated #4 Strategic Approach #2 Creative #5 Strategic Process #3 Confident ▲ problems reworded ▲ problem formats reviewed: rewording/removal of grids for answer formats ▲ new holistic scoring developed • Preliminary • Partial • Complete • Elegant	▲ dispositions gave important, useful information ▲ problem scoring positive, effective, quite consistent ▲ anecdotal records still problematic ▲ student affect responses not useful ▲ environmental scan questioned

PROBLEM SOLVING DISPOSITIONS

Motivated

- shows interest, curiosity, enthusiasm and inventiveness in doing mathematics*
- goes beyond the assignment
- reluctant to stop*
- works to a goal
- works until satisfied with solution
- works for a lengthy period using one strategy

Creative

- flexible in exploring mathematical ideas*
- tries alternative methods*
- invents own problems, formulates problems
- able to improvise in selecting materials to represent the problem

Confident

- belief in ability to use mathematics to solve problems*
- belief in ability to reason*
- not afraid to make mistakes
- belief that problems can be solved
- ability to identify and use necessary resources
- prepared to demonstrate for other students
- belief in ability to communicate ideas

Strategic Approach

- can state the problem in own words
- understands the meaning of words in the problem
- is able to draw a diagram that reflects the problem
- knows what information is needed to solve the problem and what information is extraneous**

Strategic Process

- uses organized and logical approaches
- takes time to stop and think
- notices and corrects one's mistakes**
- monitors one's thinking* – is reflective

* Terminology used in NCTM, *Curriculum and Evaluation Standards for School Mathematics*, 1989. p. 228

** Terminology used in Alberta Education, *Diagnostic Mathematics Program*, 1991, "Cognitive Processes."

These mathematical dispositions, identified by the EQI-Math project, should be used to:

i) frame the observations, i.e., to determine what aspect(s) of students' behaviour to focus on;

ii) interpret the students' actions.

The dispositions describe the type of behaviours that are associated with successful problem solving. Thus they should be used as a basis of determining whether a student possesses or does not possess these qualities. It is more useful to wait after a few observations before making interpretations. Interpretation after each observation could be misleading since a trend cannot be established with one or two observations. Only a few students (1 to 4) should be observed in a given problem solving situation.

HOLISTIC RESPONSE CRITERIA

EQI-Math
Pilot 1991-2

Preliminary Response

No attempt

Words do not reflect the problem:
- diagrams and drawings unclear or misrepresent the problem; and/or
- does not indicate which information is appropriate to finding a solution.

Begins, but fails to move toward a solution.

Partial Response

Begins the problem in a manner that could lead to a solution but fails to complete it;
- omits a significant part of the problem;
- makes a major computational error
- uses an inappropriate strategy for solving the problem.

Provides a solution, but explanation unclear;
- argument or explanation incomplete;
- diagram is inappropriate or unclear.
  (A correct answer with no evidence to support the solution must be further supported by an interview or else there is no basis on which to judge the process.)

Complete Response

Gives a complete response that includes a full solution with reasonably clear explanations;
- communicates sound supporting arguments;
- shows understanding of the mathematical ideas and processes.

Elegant Response

Gives a complete response with an "elegant" explanation;
- unusual insight displayed in solution;
- communicates strong supporting arguments.

The EQI-Math project identified these Holistic Response Criteria as a basis of interpreting and assessing written solutions.

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EDUCATION